CROSS-NATIONAL AND TEMPORAL VARIATION IN THE INTERGENERATIONAL ELASTICITY OF EXPECTED INCOME: A THEORETICAL MODEL

Pablo A. Mitnik
(pmitnik@stanford.edu)

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Abstract

This paper develops a theoretical model aimed at accounting for cross-national and temporal variation in intergenerational mobility and persistence and at identifying levers for social policy. The model allows to interpret variation in the intergenerational elasticity of expected income (IGE) in terms of differences in structural parameters. It shows that the IGE can be expressed as a function of parameters representing the productivity of private and public investments in human capital, the return to human capital, the progressivity of public investment in human capital, the degree of socioeconomic residential segregation, and the degree to which taxes and transfers reduce income inequality as measured by the Gini coefficient.

Introduction

This paper develops a theoretical model aimed at accounting for cross-national and temporal variation in intergenerational mobility and persistence and at identifying levers for social policy. Like previous models (e.g., Solon 2004; Benabou 2000; Durlauf and Seshadri 2018), it provides a theoretical framework that allows to interpret variation in intergenerational income persistence across countries and periods in terms of structural parameters. Unlike previous models in the literature, it focuses not on the intergenerational elasticity (IGE) conventionally estimated by mobility scholars but on the IGE of expected income (for recent work estimating this IGE see Mitnik et al. [2015, 2018a, 2018b], Mitnik and Grusky [2017], Mitnik [2017a, 2017b, 2017c] and Helsø [2018]). The model also takes into account a broader set of determinants of economic persistence that previous models and, in particular, than the widely-cited model advanced by Solon (2004).

The model shows that the IGE of market (i.e., pre-tax) income can be expressed as a function of parameters representing the productivity of private and public investments in human capital, the return to human capital, the progressivity of public investment in human capital, the degree of socioeconomic residential segregation, and the degree to which taxes and transfers reduce income inequality as measured by the Gini coefficient. As in the related literature, the notion of human capital used here is very broad, as it is meant to cover not only formal education, intelligence, "people skills" and the like, but also social capital, cultural capital, and adscriptive characteristics as long as they matter for success in the labor market (due to taste-based discrimination, statistical discrimination, or any other reason). Arguably, a more appropriate label would be "labor-market-relevant capital," or just "labor-market capital."

The theoretical analysis presented here is a barebones (in terms of references, arguments and interpretation), simplified and partial version of an analysis that I will report in full in a longer version of this paper that is in progress.

Basic features of the theoretical model

Each family includes a parent and a child (for simplicity of exposition, I will use female pronouns to refer to them). Each person lives two periods, childhood and adulthood. Human capital is obtained in childhood. In adulthood, each person has and raises a child of her own, consumes, works, and pays taxes or receives cash transfers. Labor income is a function of human capital. Taxes and cash transfers are a function of labor income and of the tax-and-welfare regime. The only sources of income are the labor market and public cash transfers, as there is no savings and the parent does not pass any economic assets to the child.

Human capital and labor income

A child's human capital in adulthood, K_i, is the result of five factors as follows:

$$K_i = \left[K_{pi}\, K_{ni}\, (I_i + G_i)\, \psi_i\right]^{\theta}, \qquad [1]$$

where $I_i > 0$ is the parent's investment on the child's human capital, $G_i > 0$ is the government's investment on the child's human capital, $K_{pi} > 0$ is the parent's human capital, $K_{ni} > 0$ is the average human capital in the neighborhood where the child grows up, and $\psi_i > 0$ is the children's luck, which is identically and independently distributed with mean one. The parameter $\theta > 0$ is the elasticity of human capital to the generating factors, in particular to the total (parent's plus government's) investment in the child's human capital.

The child's lifetime market or labor income, YL_i, is determined by her human capital as follows:

$$YL_i = [K_i]^{\pi}, \qquad [2]$$

where $\pi > 0$ is the labor-income return to human capital, expressed as an elasticity. Substituting [1] into [2] yields:

$$YL_i = \left[K_{pi}\, K_{ni}\, (I_i + G_i)\, \psi_i\right]^{\eta}, \qquad [3]$$

where $\eta = \theta\pi$ is (in particular) the elasticity of market income to total investment in human capital.

Similar equations apply to the parent's human capital and market income. It follows from the foregoing that both the parent's and the child's human capital and market income are positive. Without any loss of generality—as this can be achieved by simply changing the units used to measure income—I assume that market income is not just positive but equal or larger than one.

The government's roles

The government collects taxes on market income and makes cash transfers. Those above a tax threshold pay taxes while those below the threshold receive transfers. Transfers assume the form of negative taxes. Tax rates are a function of market income. Denoting the parent's market or labor income by XL_i, the disposable incomes of the parent and the child are

$$X_i = XL_i(1 - tp_i) = XL_i \left[\frac{\delta_0}{XL_i}\right]^{\delta_1} = XL_i^{\,1-\delta_1}\, \delta_0^{\,\delta_1} \qquad [4]$$

$$Y_i = YL_i(1 - tc_i) = YL_i \left[\frac{\delta_0}{YL_i}\right]^{\delta_1} = YL_i^{\,1-\delta_1}\, \delta_0^{\,\delta_1}, \qquad [5]$$

where $tp_i = 1 - \left[\frac{\delta_0}{XL_i}\right]^{\delta_1}$ and $tc_i = 1 - \left[\frac{\delta_0}{YL_i}\right]^{\delta_1}$ are, respectively, the parent's and the child's tax rates (which may be negative), $\delta_0 \geq 1$, and $0 < \delta_1 < 1$.

The tax threshold δ_0 determines who pays taxes and who receives transfers, as $\delta_0 > XL_i$ entails that $\left[\frac{\delta_0}{XL_i}\right]^{\delta_1} > 1$, so that the parent receives a transfer, while $\delta_0 < XL_i$ entails that $\left[\frac{\delta_0}{XL_i}\right]^{\delta_1} < 1$, so that the parent pays taxes. Of course, a similar analysis applies for the child (in adulthood). Jointly with the ratio between the tax threshold and actual market income, δ_1 determines how much is paid in taxes or received in transfers. Moreover, ${\delta_0}^{\delta_1}$ is the minimum disposable income a person may have in this economy.

Although δ_0 and δ_1 together define the economy's tax-and-welfare regime, how much the inequality in disposable (i.e., after-tax-and-transfers) income proportionally departs from the inequality in market (i.e., before-tax-and-transfers) income is a function of δ_1 alone, as long as we use the Gini coefficient to measure income inequality. This is so because the Gini coefficient can be expressed (e.g., Lerman and Yitzhaki 1984) as:

$$G = 2\frac{Cov(I, r(I))}{E(I)}$$

where I is an income variable and r(I) is the rank of the person based on that income variable. Therefore, the ratio between the Ginis of the two income measures is:

$$\frac{2\frac{Cov\left({XL_i}^{1-\delta_1}\,{\delta_0}^{\delta_1}, r\left({XL_i}^{1-\delta_1}\,{\delta_0}^{\delta_1}\right)\right)}{E({XL_i}^{1-\delta_1}\,{\delta_0}^{\delta_1})}}{2\frac{Cov(XL_i, r(XL_i))}{E(XL_i)}} = \frac{\frac{Cov\left({XL_i}^{1-\delta_1}, r(XL_i)\right)}{E({XL_i}^{1-\delta_1})}}{\frac{Cov(XL_i, r(XL_i))}{E(XL_i)}},$$

where I have used that the rank remains the same when the variable it is based on is multiplied by a scalar. Hence, the ratio of Gini coefficients depends on δ_1 but not on δ_0. Moreover, when $\delta_1 = 0$ the ratio is 1, meaning that there is no reduction in income inequality due to taxes and transfers; and when $\delta_1 = 1$ the ratio is 0, meaning that taxes and transfers fully eliminate income inequality.

The government also makes investments on the children's human capital (for instance, through public investments in education and health), denoted by G_i.

Lastly, the government maintains a balanced budget, that is:

$$\sum_i (XL_i - X_i) - G_i) = \sum_i \left(XL_i - {XL_i}^{1-\delta_1}\,{\delta_0}^{\delta_1} - G_i\right) = 0,$$

and does it by adjusting its investments on human capital so that:

$$\sum_i G_i = \sum_i XL_i - {\delta_0}^{\delta_1} \sum_i {XL_i}^{1-\delta_1}.$$

The parent's investment decision

The parent allocates her lifetime disposable income between own consumption, C_i, and investment in her child's human capital. The budget constraint for the parent is therefore:

$$X_i = I_i + C_i\,. \qquad [6]$$

In making her investment decision, the parent maximizes the Cobb-Douglas utility function

$$U = (1 - \omega) \ln C_i + \omega \ln(Y_i),$$

where $0 < \omega < 1$ reflects the parent's relative preference for child disposable income (against own consumption). Let's assume that (a) the parent knows Equation [3], G_i, and the tax-and-welfare system, (b) the parent believes that the tax-and-welfare system will remain unchanged in the next generation, and (c) an interior solution exists. It then follows that:

$$U = (1-\omega)\ln(X_i - I_i) + \omega \ln\left(YL_i{}^{1-\delta_1}\, \delta_0{}^{\delta_1}\right)$$
$$= (1-\omega)\ln(X_i - I_i) + \omega(1-\delta_1)\,\eta\, \ln\left[K_{pi}\, K_{ni}\, (I_i + G_i)\, \psi_i\right] + \omega\, \delta_1 \ln \delta_0.$$

The first-order condition for maximizing U is:

$$\frac{\partial U}{\partial I_i} = -\frac{(1-\omega)}{X_i - I_i} + \frac{\omega(1-\delta_1)\,\eta}{I_i + G_i} = -\frac{(1-\omega)}{XL_i{}^{1-\delta_1}\, \delta_0{}^{\delta_1} - I_i} + \frac{\omega(1-\delta_1)\,\eta}{I_i + G_i} = 0,$$

and therefore the parent's investment is:

$$I_i = \frac{\eta\, \omega\, (1-\delta_1)\, \delta_0{}^{\delta_1}}{(1-\omega) + \eta\, \omega(1-\delta_1)} XL_i{}^{1-\delta_1} - \frac{(1-\omega)}{(1-\omega) + \eta\, \omega(1-\delta_1)} G_i\,. \qquad [7]$$

Equation [7] has several implications, which are discussed in the longer version of this paper mentioned in the introduction.

IGE of expected market income with respect to parental market income

Substituting Equation [7] into Equation [3] we obtain:

$$YL_i = \left\{K_{pi}\, K_{ni}\, \left[\frac{\eta\, \omega(1-\delta_1)}{(1-\omega) + \eta\, \omega(1-\delta_1)} X_i - \frac{(1-\omega)}{(1-\omega) + \eta\, \omega(1-\delta_1)} G_i + G_i\right] \psi_i\right\}^{\eta}. \qquad [8]$$

The expression in square brackets can be written as:

$$\frac{\eta\, \omega(1-\delta_1)}{(1-\omega) + \eta\, \omega(1-\delta_1)}\left[X_i - \frac{(1-\omega)}{\eta\, \omega(1-\delta_1)} G_i + \frac{(1-\omega) + \eta\, \omega(1-\delta_1)}{\eta\, \omega(1-\delta_1)} G_i\right] =$$
$$\frac{\eta\, \omega(1-\delta_1)}{(1-\omega) + \eta\, \omega(1-\delta_1)}\left[X_i\left(1 + \frac{G_i}{X_i}\right)\right].$$

Substituting the last expression into Equation [8], and exponentiating and taking logarithm on the RHS yields:

$$YL_i = \left\{K_{pi}\, K_{ni}\, \frac{\eta\, \omega(1-\delta_1)}{(1-\omega) + \eta\, \omega(1-\delta_1)} \left[X_i\left(1 + \frac{G_i}{X_i}\right)\right] \psi_i\right\}^{\eta}$$
$$= \exp\left\{\eta\left[\ln \frac{\eta\, \omega(1-\delta_1)}{(1-\omega) + \eta\, \omega(1-\delta_1)} + \ln\left(K_{pi}\, K_{ni}\right) + \ln X_i + \ln\left(1 + \frac{G_i}{X_i}\right) + \ln \psi_i\right]\right\}. \quad [9]$$

If $\frac{G_i}{X_i}$ is small, $\ln\left(1 + \frac{G_i}{X_i}\right) \cong \frac{G_i}{X_i}$. Somewhat similarly to Solon (2004), I assume that the public policy is such that the following is approximately the case:

$$\frac{G_i}{X_i} \cong \varphi_0(\delta_0, \delta_1, \varphi_1) - \varphi_1 \ln(X_i) \geq 0,$$

where $\varphi_0(\delta_0, \delta_1, \varphi_1) > 0,\ \varphi_1 \geq 0$ and the function $\varphi_0(\delta_0, \delta_1, \varphi_1)$ satisfies the balanced-budget constraint. A positive φ_1 indicates relative progressivity, as the ratio of public investment to parental disposable income falls with parental income (absolute public investment may or may not be larger for disadvantage children). Therefore, we have:

$$\ln\left(1 + \frac{G_i}{X_i}\right) \cong \varphi_0(\delta_0, \delta_1, \varphi_1) - \varphi_1 \ln(X_i). \quad [10]$$

Now, similarly to Benabou (2018), I assume that the average human capital in the child's neighborhood is approximately

$$K_{ni} \cong \left[K_{pi}\right]^{\tau} \widetilde{K}^{1-\tau}. \qquad [11]$$

Here $\tau \in [0,1]$ measures the degree of socioeconomic residential segregation in the economy, while $\widetilde{K}$ is a function of all parents' human capital and of τ, such that the sum of the K_{ni} across children equals the average human capital per adult in the economy. More precisely:

$$\widetilde{K} = \frac{\sum_i K_{pi}}{N \sum_i \left[K_{pi}\right]^{\tau}},$$

where N is the number of adults in the economy.[1] When $\tau = 1$, the average parental capital in the neighborhood is the same as the parent's capital (complete residential segregation), while when $\tau = 0$ the average parental capital in the neighborhood is the same as the average in the economy (no segregation). For intermediate values of τ (i.e., of segregation) the average parental capital in the child's neighborhood is more or less similar to her parent's, depending on the exact value of τ.

Using the version of Equation [2] that applies to parents, and Equation [11], we may write:

$$\ln\left(K_{pi}\, K_{ni}\right) = \ln\left\{[XL_i]^{1/\pi} \left[(XL_i)^{1/\pi}\right]^{\tau} \widetilde{K}^{1-\tau}\right\}$$
$$= \frac{1+\tau}{\pi} \ln XL_i + (1-\tau)\ln \widetilde{K}. \qquad [12]$$

Substituting Equations [10] and [12] into Equation [9] yields:

$$YL_i \cong \exp\left\{\eta \left[\ln \frac{\eta\, \omega(1-\delta_1)}{(1-\omega) + \eta\, \omega(1-\delta_1)} + \frac{1+\tau}{\pi} \ln XL_i + (1-\tau)\ln \widetilde{K} + \ln X_i + \varphi_0(\delta_0, \delta_1, \varphi_1) - \varphi_1 \ln(X_i) + \ln \psi_i\right]\right\}. \qquad [13]$$

Now, from Equation [4],

$$\ln X_i = (1-\delta_1)\ln XL_i + \delta_1 \ln \delta_0.$$

Substituting this expression into Equation [13] I obtain, after some algebraic work:

[1] With $\widetilde{K}$ defined this way, $\sum_i K_{ni} = \widetilde{K}^{1-\tau} \sum_i \left[K_{pi}\right]^{\tau} = \frac{\sum_i K_{pi}}{N \sum_i \left[K_{pi}\right]^{\tau}} \sum_i \left[K_{pi}\right]^{\tau} = \frac{\sum_i K_{pi}}{N}$, as required.

$$YL_i \cong \exp\left\{\theta\,\pi\left[\ln\frac{\theta\,\pi_1\,\omega(1-\delta_1)}{(1-\omega)+\theta\,\pi\,\omega(1-\delta_1)} + (1-\tau)\ln\widetilde{K} + \varphi_0(\delta_0,\delta_1,\varphi_1) + (1-\varphi_1)\delta_1\ln\delta_0\right]\right.$$
$$\left. + [\theta(1+\tau)+\theta\,\pi\,(1-\delta_1)(1-\varphi_1)]\ln XL_i\right\}\,[\psi_i]^{\theta\pi}.$$

$$YL_i \cong \exp\left\{\theta\,\pi\left[\ln\frac{\theta\,\pi\,\omega(1-\delta_1)}{(1-\omega)+\theta\,\pi\,\omega(1-\delta_1)} + (1-\tau)\ln\widetilde{K} + \varphi_0(\delta_0,\delta_1,\varphi_1) + (1-\varphi_1)\delta_1\ln\delta_0\right.\right.$$
$$\left.\left. + \frac{\ln E([\psi_i]^{\theta\pi})}{\theta\,\pi_1}\right] + [\theta(1+\tau)+\theta\,\pi(1-\delta_1)(1-\varphi_1)]\ln XL_i\right\}\frac{[\psi_i]^{\theta\pi}}{E([\psi_i]^{\theta\pi})}. \qquad [14]$$

Therefore we may write:

$$YL_i \cong \exp(\alpha_0 + \alpha_1 \ln XL_i)\,\xi_i, \qquad [15]$$

where

$$\alpha_0 = \theta\,\pi\left[\ln\frac{\theta\,\pi\,\omega(1-\delta_1)}{(1-\omega)+\theta\,\pi\,\omega(1-\delta_1)} + (1-\tau)\ln\widetilde{K} + \varphi_0(\delta_0,\delta_1,\varphi_1) + (1-\varphi_1)\,\delta_1\ln\delta_0\right.$$
$$\left. + \frac{\ln E([\psi_i]^{\theta\pi})}{\theta\,\pi}\right]$$

$$\alpha_1 = \theta(1+\tau) + \theta\,\pi(1-\delta_1)(1-\varphi_1)$$

and $\xi_i = \frac{[\psi_i]^{\theta\pi}}{E([\psi_i]^{\theta\pi})}$ is a mean-one I.I.D. stochastic term whose value reflects the children's luck and the return to the investment on human capital.

Taking the expectation conditional on a value of parental labor income, and then taking logarithm, yields:

$$\ln E(YL|xl) \cong \alpha_0 + \alpha_1 \ln xl. \qquad [16]$$

Therefore $\alpha_1 = \frac{\partial \ln E(YL|xl)}{\partial \ln xl}$ is *the IGE of children's expected market income with respect to parents' market income*, and this IGE is greater as:

(a) The productivity of private and public investments on human capital, θ, is greater;
(b) The return to human capital, π, is greater;
(c) The relative progressivity of public investment, φ_1, is smaller;
(d) The degree of socioeconomic residential segregation, τ, is larger; and
(e) The degree to which taxes and transfers reduce income inequality as measured by the Gini coefficient is lower (as there is a one-to-one mapping between δ_1 and the ratio between the Gini coefficients for disposable and market income, with the ratio falling as δ_1 increases).

The implications for cross-country comparisons and for within-country comparisons across time, as well as the policy implications, are immediate.

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