



**TWO-SAMPLE ESTIMATION OF THE INTERGENERATIONAL  
ELASTICITY OF EXPECTED INCOME**

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## Abstract

The intergenerational income elasticity (IGE)—the workhorse measure of economic mobility—has very often been estimated with short-run income measures drawn from two independent samples and using the Two-Sample Two-Stage Least Squares estimator. The IGE conventionally estimated in the literature, however, has been widely misinterpreted: While it is assumed that it pertains to the conditional expectation of children's income, it actually pertains to its conditional geometric mean. This has led to a call to replace it by the IGE of the expectation, which requires developing the methodological knowledge necessary to estimate the latter with short-run income measures drawn from two independent samples. This paper contributes to this aim in three ways. First, it advances a two-sample Generalized Method of Moments estimator of the exponential regression model, which can be used to estimate the IGE of the expectation. Second, it develops a generalized error-in-variables model for the estimation of the IGE of the expectation with that estimator and short-run income measures. Lastly, the paper uses data from the Panel Study of Income Dynamics to estimate the IGE of the expectation with the instruments typically available to mobility scholars. The empirical results are consistent with the predictions of the formal analysis and show that, if the income measures are obtained when parents and children are close to 40 years old, the estimates generated by the two-sample estimator need to be interpreted as upper-bound estimates and vary substantially across instruments. This suggests that mobility scholars should estimate the IGE with a variety of instruments, and then select the estimate that provides the tightest bound as the preferred upper-bound estimate.

## Introduction

The intergenerational income elasticity (IGE) is the workhorse measure of economic mobility (Solon 1999; Corak 2006; Black and Devereaux 2011). It has been very extensively estimated to assess a country's overall level of income or earnings mobility, and to conduct comparative analyses of economic mobility across geographic areas, demographic groups, and time periods (e.g., Björklund and Jäntti 2000; Chadwick and Solon 2002; Hertz 2005, 2007; Aaronson and Mazumder 2008; Bloome and Western 2011), among other purposes.

In spite of the IGE's centrality in the intergenerational-mobility field, Mitnik and Grusky (2017) have recently shown that this elasticity has been widely misinterpreted. Indeed, the IGE has been construed as pertaining to the *expectation* of children's income conditional on their parents' income—as apparent, for instance, in its oft-invoked interpretation as a measure of regression to the (arithmetic) mean. However, the IGE estimated in the literature pertains to the conditional *geometric mean* of the children's income. As explained later, this not only makes all conventional interpretations of the IGE invalid but also has very deleterious methodological consequences.

Mitnik and Grusky (2017) have argued that both the conceptual and the methodological problems can be solved in a straightforward manner by simply replacing the IGE of the geometric mean—the de facto estimated IGE—by the IGE of the expectation—the IGE that mobility scholars thought they were estimating—as the workhorse intergenerational elasticity. They have also called for effectuating such replacement. This requires, however, that the methodological knowledge needed to estimate the latter IGE with short-run income variables is made available.

Indeed, as the IGE is defined in terms of the long-run incomes of parents and children,

the fact that it is almost always estimated with short-run proxy measures of income—that is, with income variables affected by substantial measurement error—has been a central issue in the mobility literature. The main concerns have been (a) the attenuation bias that may result from estimating the IGE by Ordinary Least Squares (OLS) when parental income is measured with error, (b) the amplification bias that may result from estimating the IGE by instrumental variables (IV) and Two-Stage Least Squares (TSLS) with an invalid instrument, where the latter is invalid because it is positively correlated to the error term in the population regression function of interest, (c) the lifecycle biases that may result when the differences in short-run incomes between children or between parents do not capture well the differences in their long-run incomes, and (d) the interactions between the sources of bias just mentioned. Since the mid-2000s, the literature has relied on measurement-error models that allow for lifecycle biases and subsume the classical measurement-error model as a special case. Thus, Haider and Solon (2006) advanced the generalized error-in-variables (GEiV) model for the estimation of the conventional IGE by Ordinary Least Squares (OLS), while an analogous model for its estimation with instrumental variables—labelled the “GEIV-IV model” by Mitnik (2017b)—has been implicitly invoked by mobility scholars.<sup>1</sup>

Two recent papers developed similar measurement-error models for the estimation of the IGE of the expectation, so as to make its substitution for the IGE of the geometric mean methodologically feasible. Mitnik (2017a) advanced a generalized error-in-variables model for the estimation of the IGE of the expectation with the Poisson Pseudo Maximum Likelihood (PPML) estimator (Santos Silva and Tenreyro 2006, 2011). This is the estimator employed by Mitnik et al. (2015), the first study that estimated that IGE rather than the IGE of the geometric mean. Mitnik and Grusky (2017) and Mitnik (2017a) have proposed that this estimator plays for

the IGE of the expectation the role that the OLS estimator has played for the conventional IGE.<sup>2</sup> Likewise, Mitnik (2017b) developed two alternative generalized error-in-variables models for the estimation of the IGE of the expectation with the additive-error version of the Generalized Method of Moments (GMM) IV estimator of the Poisson or exponential regression model (Mullahy, 1997; Windmeijer and Santos Silva, 1997); he also proposed that this estimator, to which I will refer as the GMM-IVP estimator, plays for the estimation of the IGE of the expectation the same role that the linear IV and TSLS estimators have played for the estimation of the conventional IGE.

These recent contributions did not address, however, a key problem that mobility scholars very often confront. Few countries count with samples in which the incomes of parents and their children are both measured in some period during adulthood.<sup>3</sup> For this reason, the IGE has very often been estimated not just with annual and other short-run proxy income measures, but with short-run income measures drawn from two independent samples and using the Two-Sample Two-Stage Least Squares (TSTSLS) estimator (Jerrim et al. 2016)<sup>4</sup>. Although the instruments typically available to mobility scholars (e.g., father's or parents' education or occupation) are very likely invalid for the reason mentioned earlier—that is, because they are endogenous—the standard interpretation has been that the resulting estimates are still useful as upper-bound estimates (e.g., Björklund and Jäntti 1997), or as upward-biased estimates that can be adjusted *ex post* to make them comparable to OLS estimates (e.g., Corak 2006: Appendix).

It is therefore important to make available a two-sample estimator of the IGE of the expectation. Building on existing two-step IV estimators (Mullahy 1997:590-591; Carroll et al. 2006:139-140), and on the approach of stacking moment conditions first advanced by Newey (1984), I propose here a two-sample GMM estimator of the Poisson or exponential regression

model, to which I refer as the GMM-E-TS estimator, which can be used with that purpose. If the samples included information on long-run income, this would be all that is needed. However, as Jerrim et al. (2016) have pointed out in their criticism of the formal analysis underpinning the standard interpretation of the TSTSLs estimates of the conventional IGE as upper-bound estimates, the samples available only have information on the short-run (typically, annual) incomes of parents and children. To account for this fact, I also specify a generalized error-in-variables model for use with the GMM-E-TS estimator; this allows me to make explicit the full sets of conditions under which consistent, approximately consistent and upper-bound estimation of the IGE of the expectation can be achieved. The GMM-E-TS estimator and the GEiVE-TS model are the first two contributions of the paper.

The approach I advance for the two-sample estimation of the IGE of the expectation with short-run income variables is predicated on several assumptions being at least approximately correct, so it is important to assess its empirical performance. In addition, the measurement-error model has clear empirical implications for how the use of income measures pertaining to when parents or children are too young or too old to represent lifetime differences well should affect IGE estimates, and these can be tested. Here I rely on a U.S. sample from the Panel Study of Income Dynamics (PSID) to carry out the needed empirical analyses. My results indicate that the estimator works as expected, and that the observed lifecycle and amplification biases are in close agreement with the qualitative predictions of the formal analysis. They also show that, as long as the income measures are obtained when parents and children are close to 40 years old, the estimates generated with the two-sample estimator and the instruments typically available to mobility scholars need to be interpreted as upper-bound estimates (as expected), and vary significantly across instruments. These empirical conclusions are the paper's third contribution.

The structure of the rest of the paper is as follows. I first explain why the conventional IGE pertains to the conditional geometric mean of children's income rather than to its conditional expectation, as well as Mitnik and Grusky's (2017) proposal to redefine the IGE used as the workhorse measure of economic mobility. The next three sections introduce the two-sample GMM estimator of the Poisson or exponential regression model and discuss the issues involved in estimating the IGE of the expectation with short-run income variable in the two-sample context. This is followed by the empirical analyses. The last section draws the main conclusions of the paper.

### **The IGE of what? Redefining the workhorse intergenerational elasticity**

As already indicated, the conventionally estimated IGE has been widely misinterpreted. While mobility scholars have interpreted it as the elasticity of the expectation of children's income or earnings conditional on parental income, that IGE pertains in fact to the conditional geometric mean. Closely following Mitnik and Grusky's (2017) analysis, the standard population regression function (PRF) posited in the literature, which assumes the elasticity is constant across levels of parental income, is:

$$E(\ln Y | x) = \beta_0 + \beta_1 \ln x, \quad [1]$$

where  $Y$  is the child's long-run income or earnings,  $X$  is long-run parental income or father's earnings,  $\beta_1$  is the IGE as specified in the literature, and I use expressions like " $Z|w$ " as a shorthand for " $Z|W = w$ ." The parameter  $\beta_1$  is not, in the general case, the elasticity of the conditional expectation of the child's income. This would hold as a general result only if  $E(\ln Y | x) = \ln E(Y | x)$ . But, due to Jensen's inequality, the latter is not the case. Instead, as  $E(\ln Y | x) = \ln \exp E(\ln Y | x)$ , and  $GM(Y | x) = \exp E(\ln Y | x)$ , Equation [1] is equivalent to

$$\ln GM(Y | x) = \beta_0 + \beta_1 \ln x, \quad [2]$$

where GM denotes the geometric mean operator. Therefore,  $\beta_1$  is the elasticity of the conditional geometric mean, i.e., the percentage differential in the geometric mean of children's long-run income with respect to a marginal percentage differential in parental long-run income.<sup>5</sup>

As the geometric mean is undefined whenever an income distribution includes zero in its support, the IGE is undefined as well when this is the case. Mitnik and Grusky (2017:Sections III and IV) have argued that this has serious methodological consequences: It (a) makes it impossible to determine the extent to which parental economic advantage is transmitted through the labor market among women (as many women have zero earnings), and (b) greatly hinders research on the role that marriage plays in generating the observed levels of intergenerational persistence in family income (as many people remain single or have nonworking spouses, and therefore cannot be included in analyses examining the relationship between people's parental income and the income contributed by their spouses). As a result, the study of gender and marriage dynamics in intergenerational processes has been badly hampered. Equally important, Mitnik and Grusky (2017) have shown that, as a consequence of mobility scholars' expedient of dropping children with zero earnings from samples (to address what is perceived as the problem of the logarithm of zero being undefined), estimation of earnings IGEs with short-run proxy earnings measures is almost certainly affected by substantial selection biases. This makes the use of the IGE of men's individual earnings as an index of economic persistence and mobility in a country a rather problematic practice.<sup>6</sup>

To address these problems, Mitnik and Grusky's (2017) have called for redefining the workhorse measure of economic mobility. This entails replacing the PRF of Equation [1] by a PRF whose estimation delivers estimates of the IGE of the expectation in the general case. Under the assumption of constant elasticity, that PRF can be written as:



$$\ln E(Y|x) = \alpha_0 + \alpha_1 \ln x, \quad [3]$$

where  $Y \geq 0$ ,  $X > 0$  and  $\alpha_1 = \frac{d \ln E(Y|x)}{d \ln x}$  is the percentage differential in the expectation of children's long-run income with respect to a marginal percentage differential in parental long-run income. Crucially, (a) all interpretations incorrectly applied to the conventional IGE are correct under this formulation (see Mitnik and Grusky 2017:Section IV.A), and (b) the IGE of the expectation is fully immune to the methodological problems affecting the IGE of the geometric mean and, in particular, is very well suited for studying the role of marriage in the intergenerational transmission of advantage (see Mitnik and Grusky 2017:Section IVb for details; and Mitnik et al. 2015:64-68 for an empirical application).

### **A two-sample two-step estimator of the Poisson or exponential regression model**

In the two-sample context, the “main sample” has the children's income information, the “auxiliary sample” has the parents' income information, and both samples have a common set of variables (e.g., father's or parents' education or occupation) that may be used as instruments or predictors for the parents' income information. The two-sample GMM estimator I propose here is based on a two-sample two-step estimator, which in turn builds on existing one-sample two-step IV estimators. In the one-sample context, two types of two-step IV estimators of the Poisson or exponential regression model—or of broader classes of models including it—have been advanced. Predictor-substitution estimators (e.g., Mullahy 1997:590-591; Carroll et al. 2006:139-140) substitute predicted values of the endogenous variables, obtained in the first step, into the second-step equation (the exponential or Poisson regression model). Residual-inclusion estimators (e.g., Wooldridge 1999:Sec. 6.1; Terna et al. 2008) add the residuals from the first step as additional covariates in the second step. Terna et al. (2008) have shown that the latter estimators are generally consistent while the former are not. Unfortunately, residual-inclusion

estimators cannot be used in the two-sample context, as the residuals from the first step do not pertain to the observations employed by the second-step estimator. Therefore, I focus here on the predictor-substitution approach and carefully establish the assumptions needed for consistency.

The one-sample IV estimators proposed by Mullahy (1997) and Carroll et al. (2006) are derived under partially different assumptions (more on this below) but they involve identical steps and therefore produce identical estimates.<sup>7</sup> Operationally, they simply extend the approach used by the TSLS estimator of the linear regression model to the estimation of the exponential regression model. Thus, when there is only one endogenous variable, the first step estimates a conditional expectation model of that variable as a linear function of the instruments and any right-side exogenous variables included in the second step, while the second step estimates the exponential regression model of interest using the PPML estimator, with the endogenous variable replaced by its predicted values (which are computed with the parameters estimated in the first step). Two observations are important here. First, it is apparent that the two steps just described can be carried out unproblematically in the two-sample context, thus yielding a two-sample predictor-substitution estimator. Second, as IV estimation is consistent even if the instrumented variables are measured without error, the same two-step approach can be used to estimate the IGE of the expectation when the long-run income variables of children and parents are available (although in different samples). In the rest of this section I assume that this is the case. This allows me to focus exclusively on the issues generated by the fact that the information is split between two samples, while the additional complications generated by the fact that most available datasets only have short-run income measures are addressed in the next section.

*The two-sample two-step estimator with a valid instrument*

I discuss next the assumptions under which the two-sample predictor-substitution estimator of the exponential regression model is consistent or approximately consistent when all variables are measured without error. Without any loss of generality, I assume in what follows that  $E(Y) = E(\ln X) = 1$ .<sup>8</sup>

Rewriting Equation [3] in additive-error form, the PRF of interest is

$$Y = \exp(\alpha_0 + \alpha_1 \ln X) + \Psi, \quad [4]$$

where  $E(\Psi|x) = 0$ . Assuming for simplicity that there is only one quantitative instrument denoted by  $T$  (e.g., years of parental education), the first-step equation is the population linear projection:

$$\ln X = \gamma_0 + \gamma_1 T + R. \quad [5]$$

Consider now the following assumptions, which apply to all  $t$  when relevant:

$$A1. E(R|t) = 0$$

$$A2. \forall c > 0, E(\exp(cR)|t) = E(\exp(cR))$$

$$A2'. Var(R|t) = Var(R)$$

$$A3. \gamma_1 \neq 0$$

$$A4. E(\Psi|t) = E(\Psi).$$

Assumptions A1 and A3 entail that the expectation of the logarithm of parental income conditional on the value of  $T$  is a linear function of that value, while assumptions A3 and A4 entail that  $T$  is a valid instrument. These three assumptions are identical or functionally equivalent to assumptions made by Mullahy (1997) and Carroll et al. (2006) in deriving their two-step IV estimators. Assumption A2 is similar to an assumption made by Mullahy (1997), and to the standard assumption made for the estimation of Poisson models with unobserved

heterogeneity (e.g., Winkelmann 2008); it is automatically satisfied if the error in the first-step equation is independent from the instrument (rather than just mean independent, as specified by assumption A1). Assumption A2' posits that this error is homoscedastic. As I show below, A2' is an alternative to A2. Although A2' is not strictly weaker than A2 (neither assumption entails the other), the fact that the dependent variable in the first-step equation is the logarithm of an income variable may make A2' more attractive than A2.

I start by showing that under assumptions A1, A2, A3 and A4 the two-sample estimator of  $\alpha_1$  is consistent, while it is approximately consistent if A2 is replaced by A2'. Substituting Equation [5] into Equation [4], and using A1 and A3, yields:

$$Y = \exp(\alpha_0 + \alpha_1 E(\ln X | T) + \alpha_1 R) + \Psi$$

$$E(Y|t) = \exp(\alpha_0 + \alpha_1 E(\ln X | t)) E(\exp(\alpha_1 R)|t) + E(\Psi|t). \quad [6]$$

Using now A2, Equation [6] reduces to:

$$E(Y|t) = \exp(\alpha'_0 + \alpha_1 E(\ln X | t)) + E(\Psi|t), \quad [7]$$

where  $\alpha'_0 = \alpha_0 + \ln E(\exp(\alpha_1 R))$ ; and it further reduces to

$$E(Y|t) = \exp(\alpha'_0 + \alpha_1 E(\ln X | t)) \quad [8]$$

if assumption A4 also holds, that is, if the instrument is valid.<sup>9</sup> If the variable  $E(\ln X | T)$  were available in the estimation sample, Equation [8] would be consistently estimated by the PPML estimator (e.g., Santos Silva and Tenreyro 2006). Under a standard identification condition for two-step M-estimators (e.g., Wooldridge 2002:354), the PPML estimator that replaces  $E(\ln X | T)$  by estimates obtained in the first step, is also consistent.<sup>10</sup>

An alternative justification for this estimator as “approximately consistent”—which is in the spirit of the “regression calibration approximation” employed by Carroll et al. (2006) to

derive their two-step estimator—is as follows. Going back to Equation [6], a second-order Taylor-series approximation around  $E(R|t)$  gives:

$$\begin{aligned} E(\exp(\alpha_1 R)|t) &\cong \exp(\alpha_1 E(R|t)) + 0.5 \exp(\alpha_1 E(R|t)) [\alpha_1]^2 \text{Var}(R|t) \\ &\cong 1 + 0.5 [\alpha_1]^2 \text{Var}(R|t), \end{aligned}$$

where I have used assumption A1. Under assumption A2', we then obtain:

$$E(Y|t) \cong \exp(\alpha_0'' + \alpha_1 E(\ln X |t)) + E(\Psi|t), \quad [7']$$

where  $\alpha_0'' = \alpha_0 + \ln(1 + 0.5 [\alpha_1]^2 \text{Var}(R))$ ; and we obtain

$$E(Y|t) \cong \exp(\alpha_0'' + \alpha_1 E(\ln X |t)), \quad [8']$$

if A4 also holds. Equation [8'] indicates that the two-sample two-step estimator is approximately consistent for  $\alpha_1$  when A2's is substituted for A2.<sup>11</sup>

Resorting to Taylor-series expansions, Mitnik (2017a:14 and Appendix, A) has advanced an approximated closed form expression for  $\alpha_1$  in PRFs like [8] and [8']. Applying it here for future reference, it yields:

$$\alpha_1 \cong C_{\alpha_1} - \left[ (C_{\alpha_1})^2 - V_{\alpha_1} \right]^{\frac{1}{2}}, \quad [9]$$

where

$$V_{\alpha_1} = 2 [\text{Var}(E(\ln X |T))]^{-1} \quad [9a]$$

$$C_{\alpha_1} = [\text{Cov}(Y, E(\ln X |T))]^{-1}. \quad [9b]$$

*The two-sample two-step estimator with an invalid instrument*

Let's now assume that estimation is not based on the valid instrument  $T$  but on the invalid instrument  $\mathbf{T}$ , and that although A4 does not hold it is the case that:

$$A3'. \quad \gamma_1 > 0$$

$$A4'. \quad \text{Cov}(\Psi, \mathbf{T}) > 0.$$

(Throughout I use bold font to indicate that a parameter, expression or variable pertains to the analysis with the invalid instrument.) As  $\boldsymbol{\gamma}_1 > 0$  if and only if  $\ln X$  and  $\boldsymbol{T}$  are positively correlated, in the “long-run context” A3’ and A4’ are equivalent to the standard assumption that the (invalid) instruments typically available to mobility scholars are positively correlated with the logarithm of parental income and with the error term of the PRF of interest.

In order to determine the implications of A3’ and A4’, it is useful to rewrite the counterparts to Equations [7] and [7’] as follows:

$$E(\boldsymbol{Y}|\boldsymbol{t}) = \exp(\boldsymbol{\alpha}'_0 + \alpha_1 E(\ln X | \boldsymbol{t})) \quad [10]$$

$$E(\boldsymbol{Y}|\boldsymbol{t}) \cong \exp(\boldsymbol{\alpha}''_0 + \alpha_1 E(\ln X | \boldsymbol{t})), \quad [10']$$

where  $\boldsymbol{Y} \equiv Y - E(\Psi|\boldsymbol{T})$ . Now, making use again of the approximated closed-form expression introduced above, we may write:

$$\alpha_1 \cong \boldsymbol{C}_{\alpha_1} - \left[ (\boldsymbol{C}_{\alpha_1})^2 - \boldsymbol{V}_{\alpha_1} \right]^{\frac{1}{2}}, \quad [11]$$

where

$$\boldsymbol{V}_{\alpha_1} = 2 [\text{Var}(E(\ln X | \boldsymbol{T}))]^{-1} \quad [11a]$$

$$\begin{aligned} \boldsymbol{C}_{\alpha_1} &= [\text{Cov}(\boldsymbol{Y}, E(\ln X | \boldsymbol{T}))]^{-1} \\ &= [\text{Cov}(Y, E(\ln X | \boldsymbol{T})) - \text{Cov}(E(\Psi|\boldsymbol{T}), E(\ln X | \boldsymbol{T}))]^{-1} \\ &= [\text{Cov}(Y, E(\ln X | \boldsymbol{T})) - \boldsymbol{\gamma}_1 \text{Cov}(E(\Psi|\boldsymbol{T}), \boldsymbol{T})]^{-1} \\ &= [\text{Cov}(Y, E(\ln X | \boldsymbol{T})) - \boldsymbol{\gamma}_1 \text{Cov}(\Psi, \boldsymbol{T})]^{-1}.^{13} \quad [11b] \end{aligned}$$

Actual estimation, however, is not based on  $\boldsymbol{Y}$  but on  $Y$ , which is equivalent to making  $\boldsymbol{\gamma}_1 \text{Cov}(\Psi, \boldsymbol{T}) = 0$ . As assumptions A3’ and A4’ entail that  $\boldsymbol{\gamma}_1 \text{Cov}(\Psi, \boldsymbol{T}) > 0$ , while it is the case that  $\frac{\partial \alpha_1}{\partial \boldsymbol{C}_{\alpha_1}} < 0$  (see Mitnik 2017a:16 and Appendix, B), it follows that the probability limit of the

two-sample two-step estimator of the IGE of the expectation with the invalid instruments typically available to mobility scholars is larger than the true parameter. This is the same conclusion that is obtained for the conventional IGE when the latter is estimated with the TSTSLS estimator, also under the assumption that the samples have information on long-run rather than short-run income.<sup>14</sup>

### **Two-sample estimation of the IGE of the expectation with short-run income variables**

In the vast majority of cases in which it is necessary to use the two-sample approach, only short-run income information is available. This means that the income variables are affected by measurement error, which in turn entails that the assumptions for consistency (and approximate consistency) discussed in the previous section do not suffice. Even if the measurement errors were classical in nature, further empirical assumptions regarding those errors' relationships with the instruments would need to hold for the two-sample two-step estimator to be consistent. And the errors are not classical in the general case. As income- and earnings-age profiles differ across economic origins, using proxy measures taken when parents or children are too young or too old to represent lifetime differences well may result in "lifecycle biases" (e.g., Jenkins 1987; Harder and Solon 2006; Black and Devereux 2011; Mazumder 2005; Niborn and Stuhler 2016) that cannot be accounted for within the framework of the classical measurement-error model.

Let  $Z_l \geq 0$  be the children's income at age  $l$  and  $S_k > 0$  be the parents' income at age  $k$ ; without any loss of generality I assume that  $E(Z_l) = E(S_k) = 1$ .<sup>15</sup> The first-step equation is the population linear projection

$$\ln S_k = \tilde{\gamma}_0 + \tilde{\gamma}_1 D + Q_k,$$

where  $D$  is a generic instrument, i.e., an instrument that may or may not be valid. To simplify the presentation (more on this later), let's assume:

$$A5. E(Q_k|d) = 0$$

$$A6. \tilde{\gamma}_1 \neq 0.$$

Under these assumptions, the second-step equation may be written as:

$$E(Z_l|d) = \tilde{\alpha}_0 + \tilde{\alpha}_1 E(\ln S_k |d).$$

Then, using again the approximated closed-form expression employed before, we have:

$$\tilde{\alpha}_1 \cong C_{\tilde{\alpha}_1} - \left[ (C_{\tilde{\alpha}_1})^2 - V_{\tilde{\alpha}_1} \right]^{\frac{1}{2}}, \quad [12]$$

where

$$V_{\tilde{\alpha}_1} = 2 \left[ \text{Var}(E(\ln S_k |D)) \right]^{-1} \quad [12a]$$

$$C_{\tilde{\alpha}_1} = \left[ \text{Cov}(Z_l, E(\ln S_k |D)) \right]^{-1}.^{16} \quad [12b]$$

In this section I specify a set of measurement-error and lifecycle assumptions under which  $\tilde{\alpha}_1$  is equal to  $\alpha$  when the instrument is valid, and is larger than  $\alpha$  with the instruments typically available. To this end I advance a generalized error-in-variables model for the estimation of the IGE of the expectation in the two-sample context, or GEiVE-TS model. In addition to making the needed measurement-error assumptions explicit, this model allows to determine the potential effects of lifecycle biases.

#### *Assumptions of the GEiVE-TS model*

In order to introduce the assumptions of the GEiVE-TS model, it is necessary to first introduce the following population linear projections:

$$Z_l = \theta_{0l} + \theta_{1l} Y + W_l \quad [13]$$

$$\ln S_k = \pi_{0k} + \pi_{1k} \ln X + P_k, \quad [14]$$



where  $Z_l$  and  $Y$  are as defined earlier;  $\theta_{0l} + W_l$  is the (additive) measurement error in the short-run measure as a proxy for the long-run measure when  $\theta_{1l} = 1$ ;  $\theta_{1l}$  captures left-hand lifecycle bias and thus may be different from one and varies with  $l$ ;  $S_k$  and  $X$  are as defined earlier;  $\pi_{0k} + P_k$  is the (additive) measurement error in the logarithm of the short-run measure as a proxy for the logarithm of the long-run measure when  $\pi_{1k} = 1$ ; and  $\pi_{1k}$  captures right-hand lifecycle bias and thus may be different from one and varies with parents' age.<sup>17</sup>

The empirical assumptions of the GEiVE-TS model are the following:

$$M1. Cov(W_l, D) = 0$$

$$M2. E(P_k|d) = E(P_k).$$

These assumptions are similar, but not identical, to those made by the available generalized error-in-variables models for the IV estimation of IGEs—both the IGE of the geometric mean and the IGE of the expectation—in the one-sample context (see Mitnik 2017b). The fact that Equation [14] is a linear projection entails that  $E(P_k) = 0$ , so it follows that  $E(P_k|d) = 0$ . Assumptions M1 and M2 are expected to hold imperfectly but still as good approximations, at least when  $\theta_{1l} \cong \pi_{1k} \cong 1$ .

*Conditions for consistency and upward inconsistency of the short-run estimator*

In what follows I omit the subscripts  $l$  and  $k$  to simplify the notation. Using Equations [13] and [14], Equations [12a] and [12b] may be written as:

$$\begin{aligned} V_{\tilde{\alpha}_1} &= 2 [Var(\pi_1 E(\ln X|D) + E(P|D))]^{-1} \\ C_{\tilde{\alpha}_1} &= [\theta_1 Cov(Y, E(\ln S|D)) + Cov(W, E(\ln S|D))]^{-1} \\ &= [\theta_1 \pi_1 Cov(Y, E(\ln X|D)) + \theta_1 Cov(Y, E(P|D)) + \tilde{\gamma}_1 Cov(W, D)]^{-1}. \end{aligned}$$

Under assumptions M1 and M2,  $C_{\alpha_1}$  and  $V_{\alpha_1}$  reduce to:

$$V_{\tilde{\alpha}_1} = 2 [(\pi_1)^2 Var(E(\ln X|D))]^{-1} \tag{15a}$$

$$C_{\tilde{\alpha}_1} = [\theta_1 \pi_1 \text{Cov}(Y, E(\ln X|D))]^{-1}. \quad [15b]$$

If, in addition,  $\pi_1 = \theta_1 = 1$ ,  $V_{\alpha_1}$  and  $C_{\alpha_1}$  further reduce to

$$V_{\tilde{\alpha}_1} = 2 [\text{Var}(E(\ln X|D))]^{-1}. \quad [16a]$$

$$C_{\tilde{\alpha}_1} = [\text{Cov}(Y, E(D))]^{-1}. \quad [16b]$$

Before discussing the implications of Equations [12], [16a] and [16b], I will show that, if the assumptions of the GEiVE model hold and  $\pi_1 = \theta_1 = 1$ , then:

- (a) Assumption A1 entails assumption A5, and
- (b) Each of assumptions A3 and A3' entails A6.

First, using that  $E(\ln S) = E(\ln X) = 1$  and  $\pi_1 = 1$  entail that  $\pi_0 = 0$ , that assumption M2 implies that  $\text{Cov}(P, D) = 0$ , and Equation [14], we have:

$$\tilde{\gamma}_1 = \frac{\text{Cov}(\ln S, D)}{\text{Var}(D)} = \frac{\pi_1 \text{Cov}(\ln X, D) + \text{Cov}(P, D)}{\text{Var}(D)} = \frac{\text{Cov}(\ln X, D)}{\text{Var}(D)} = \gamma_1$$

$$\tilde{\gamma}_0 = E(\ln S) - \tilde{\gamma}_1 E(D) = \pi_0 + \pi_1 E(\ln X) - \tilde{\gamma}_1 E(D) = E(\ln X) - \gamma_1 E(D) = \gamma_0.$$

As  $\tilde{\gamma}_1 = \gamma_1$ , each of A3 and A3' guarantees that A6 holds as well. Second, we may now write

$$\ln S = \gamma_0 + \gamma_1 D + Q = \ln X + P,$$

which indicates that  $R = Q - P$ . Then, as  $E(P|d) = 0$  (per assumption M2), it follows that assumption A1,  $E(R|d) = 0$ , guarantees that  $E(Q|d) = 0$  as well. Therefore, if both M1 and M2 hold and  $\pi_1 = \theta_1 = 1$ , Equations [16a] and [16b] follow as long as A1 and either A3 or A3' hold.

Now, let's assume that Equations A1, A2 or A2', A3 and A4 hold, that is, let's consider the case in which the instrument is valid. Comparing Equations [12], [16a] and [16b] with Equations [9], [9a] and [9b] makes clear that in this scenario the "short-run estimator" (the two-sample two-step estimator with short-run income variables) is a consistent (or approximately

consistent) estimator of the IGE of the expectation as long as (a) the measurement-error assumptions  $Cov(W, T) = 0$  and  $E(P|t) = E(P)$  hold, and (b) the short-run variables pertain to the “right points” of the children’s and parents’ lifecycles, that is, they pertain to when  $\pi_1 = \theta_1 = 1$  (more on this below).

Let’s consider next the case in which the instrument is invalid because A4 does not hold, but A3’ and A4’ do hold. Comparing now Equations [12], [16a] and [16b] with Equations [11], [11a] and [11b] shows that, under the same measurement-error and lifecycle assumptions, the short-run estimator is upward inconsistent with the invalid instruments typically available to mobility scholars.<sup>18</sup>

### *Lifecycle biases*

As indicated earlier, using proxy income measures taken when parents or children are too young or too old to represent lifetime differences well may result in lifecycle biases. When  $\pi_1$  and  $\theta_1$  are not equal to one, Equations [15a] and [15b] still hold if we assume that A5 and A6 hold.<sup>19</sup> For the purposes here, it is more convenient to write Equation [12] and those two equations as follows:

$$\tilde{\alpha}_1 \cong \frac{1}{\pi_1} \left\{ C_{\tilde{\alpha}_1} - \left[ (C_{\tilde{\alpha}_1})^2 - V_{\tilde{\alpha}_1} \right]^{\frac{1}{2}} \right\}, \quad [17]$$

where

$$V_{\tilde{\alpha}_1} = 2 [Var(E(\ln X|D))]^{-1} \quad [17a]$$

$$C_{\tilde{\alpha}_1} = [\theta_1 Cov(Y, E(\ln X|D))]^{-1}. \quad [17b]$$

With the equations written this way, it is easy to see the implications of the GEiVE-TS model for the lifecycle biases that may affect the estimation of the IGE with short-run income variables.

Indeed, income-age profiles vary in a well-known manner across people with different levels of human capital, which leads to the hypothesis that  $\theta_1$  and  $\pi_1$  will increase with children's and parents' ages, respectively, and that they will be smaller than one when the children or parents are younger and larger than one when they are older (Harden and Solon 2006). Moreover, this hypothesis has received empirical corroboration both for  $\pi_1$  (Harden and Solon 2006; Nybom and Stuhler 2016; Mitnik 2017a and 2017b) and  $\theta_1$  (Mitnik 2017a), with the “measurement-error slopes” approaching the value one when the parents and children, respectively, are close to 40 years old. Using this information, a comparison of Equations [17], [17a] and [17b] with Equations [9], [9a] and [9b], and with Equations [11], [11a] and [11b], allows to predict the directions of the lifecycle biases.

To simplify the analysis—and as Solon and Harder (2006) did in their analyses of lifecycle biases in the OLS estimation of the IGE of the geometric mean—I will consider the GEiVE-TS model's implications regarding left-side and right-side lifecycle biases separately. Thus, I first assume that  $\pi_1 = 1$ , so as to focus on left-side lifecycle bias exclusively. As  $\frac{\partial \tilde{\alpha}_1}{\partial c_{\tilde{\alpha}_1}} < 0$  and  $\frac{\partial c_{\tilde{\alpha}_1}}{\partial \theta_1} < 0$ , it follows that the short-run estimator can be expected to be downward inconsistent when the children are too young (i.e., when  $\theta_1 < 1$ ) and upward inconsistent when they are too old (i.e., when  $\theta_1 > 1$ ), as long as the instrument is valid. With the instruments typically available, using measures of children's income pertaining to when they are older can be expected to exacerbate the amplification bias associated to the use of invalid instruments, while using measures pertaining to when they are younger can be expected to reduce that amplification bias (if  $\theta_1$  is small enough, the use of these measures could even more than compensate for the upward bias generated by the invalid instruments).

I now assume that  $\theta_1 = 1$ , so as to focus on right-side lifecycle bias exclusively. It then follows that, with a valid instrument,  $\tilde{\alpha}_1 = \frac{\alpha_1}{\pi_1}$ , that is, that the probability limit of the short-run estimator is  $\frac{1}{\pi_1}$  times the value of the true parameter. This means that we can expect the estimator to be upward inconsistent when the parents are too young (i.e., when  $\pi_1 < 1$ ) and downward inconsistent when they are too old (i.e., when  $\pi_1 > 1$ ). With the invalid instruments typically available, the probability limit of the estimator when the parents are too young will be larger than  $\frac{1}{\pi_1}$  times the true parameter, thus exacerbating the amplification bias associated to the use of invalid instruments. When parents are too old, the probability limit will be smaller than  $\frac{1}{\pi_1}$  times the true parameter (in fact, if  $\pi_1$  is large enough, this may more than compensate for the amplifying effect of using an invalid instrument).

More generally, using measures of parents' income pertaining to when they are younger (older) should *contribute* to overestimating (underestimating) the IGE of expected income, while using measures of children's income pertaining to when they are younger (older) should *contribute* to underestimating (overestimating) that IGE. Whether the probability limit of the short-run estimator is actually larger or smaller than the true parameter will in each case depend on the values of both measurement-error slopes and on whether the instrument is valid or not. In particular, the standard conclusion that two-sample IGE estimates are upward biased (given the instruments typically available) only follows if the short-run income variables satisfy some conditions: The parental variable has to pertain to an age at which  $\pi_1 \leq 1$ , while the children variable has to pertain to an age at which  $\theta_1 \geq 1$ . Given an invalid instrument, the optimal upper-bound estimates in term of tightness, among those that the GEIV-TS model predicts will be upper biased, can be expected to be those obtained when  $\pi_1 = \theta_1 = 1$ .

## **Transforming the two-sample two-step estimator into a two-sample GMM estimator**

In the one-sample context, and following the approach first advanced by Newey (1984), a two-step estimator—where the estimator in the second step is itself an M-estimator or a GMM estimator that depends on the first-step estimator—can be easily transformed into a GMM two-equation estimator (where the two equations are estimated simultaneously). To do so, the first-order conditions for the two equations are “stacked,” so that the first-order conditions for the full GMM problem reproduce the first-order conditions of the estimators employed in each step.

There are two main advantages to using this approach. First, the GMM estimator only involves standard asymptotic inferential procedures, while the two-step estimator requires to account for the two-step nature of the estimation by using more complicated closed-form asymptotic variance estimators (e.g., Murphy and Topel 1985; Hardin 2002), or resampling methods. Second, transforming the two-step estimator into a GMM estimator ensures efficient estimation (see Wooldridge 2002:425 and ff. for more details). As efficiency is achieved by weighting instruments in an optimal way, in finite samples the two-step and GMM estimators will produce identical estimates when there is only one instrument, but will generally produce somewhat different estimates when there are multiple instruments.

With a minor modification, the same approach can be used to transform the two-sample two-step predictor-substitution estimator of the exponential regression model introduced above into a two-sample two-equation GMM estimator.<sup>20</sup> Let’s stipulate that the “first equation” is the equation from the first step, and the “second equation” is the equation from the second step. Then a two-sample GMM estimator of the IGE of the expectation, where the moment conditions are products of instruments and “modified residuals,” is obtained as follows: (a) replace the missing information in each of the samples—the logarithm of parents’ income in the main

sample, children's income in the auxiliary sample—by any value, e.g., zero, (b) stack the data from the two samples into one sample, adding an indicator variable to identify the observations from the auxiliary sample, (c) define the modified residuals entering the moment conditions associated to the first equation as the usual residuals multiplied by the indicator variable, (d) define the modified residuals entering the moment conditions associated to the second equation as the usual residuals multiplied by one minus the indicator variable, and (e) estimate the two-equation model by GMM in the usual way.

The key steps are (c) and (d). The modified residuals defined in those steps are equal to the usual residuals for the observations in the equation-dependent “relevant sample” but are always equal to zero—regardless of the value of the parameter vector—for the observations in the equation-dependent “irrelevant sample.” Therefore, estimation can proceed as in the one-sample context. The resulting estimator is the GMM-E-TS estimator.

### **Empirical analyses**

In this section I empirically assess the performance of the two-sample estimation approach advanced in the previous sections, test the predictions of the measurement-error model regarding lifecycle biases, and provide evidence that two-sample estimates of the IGE of the expectation are, as expected, upward biased when the short-run variables pertain (a) to the ages at which the measurement-error slopes are exactly one, and (b) to when parents and children are close to 40 years old. I also provide empirical information on the absolute and relative magnitudes of the resulting biases. The empirical analyses are preceded by a brief description of the data used and of some estimation details.

### *Data and estimation*

The empirical analyses are based on a PSID sample that makes it possible to construct an approximated measure of long-run parental income but not of children's long-run income.<sup>21</sup> However, as I explain below, this sample still allows to shed light on the key questions of interest for this paper. The sample includes information on children born between 1966 and 1974, for which 25 years of parental data centered on age 40 (pertaining to when the children were between 1 and 25 years old) are available. Children observed in the PSID when they were between 35 and 38 years old are included in the sample. I use information on the average family income of children when they were 35-38 years old, on parents' family income, age, and years of education when the children were 1-25 years old, and on fathers' occupation when the children were growing up (as reported by the latter). I do not use information on individual earnings because I only estimate the IGE of family income. Table 1 presents descriptive statistics, while the Appendix provides additional details on the sample and variables and explains why I focus exclusively on the family-income IGE.

In order to conduct two-sample analyses, I proceed as follows: (a) I duplicate the PSID sample; (b) I replace parental income values by missing values in the original sample, which I use as the main sample; and (c) I replace children's income values by missing values in the second sample, which I use as the auxiliary sample. Proceeding this way is very advantageous, as it allows to address the questions of interest for this paper cleanly, without having to worry about the effects of the data not satisfying the "common-population assumption," or of sheer sampling variability even when that assumption is satisfied.<sup>22</sup>

I use the PPML and GMM-IVP estimators to estimate the IGE of the expectation in the one-sample context, and the GMM-E-TS estimator to estimate it in the two-sample context.<sup>23</sup> I



employ the following instruments when estimation is based on the GMM-IVP or the GMM-E-TS estimators: (a) parents' total years of education when the child was 13 years old, (b) the household head's years of education when the child was 13 years old, (c) the father's occupation, and (d) the father's occupation and either the parents' or the household head's education.<sup>24</sup> Reiss (2016) has shown that, in IV estimation, the functional form chosen for the instruments may be very consequential in some cases, so I also use second-degree polynomials on the parental education variables as instruments. I employ sampling weights and compute cluster-robust standard errors in all cases.<sup>25</sup>

The relationship between long-run and short-run income measures varies with the age at which income is measured; for this reason, it is customary to include polynomials on children's and parents' ages as controls when estimation is based on short-run measures. However, as all IGE estimates I report pertain to a sample in which the variation in children's ages is very small, controlling for children's age is unnecessary (see the next paragraph for a second reason for proceeding this way). Mitnik et al. (2015:34) have argued that the age at which parents have their children is not exogenous to their income, that parental age is causally relevant for their children's life chances, and that insofar as we want persistence measures to reflect the gross association between parental and children's income we should not control for parental age. Here I present estimates from models without controls for parental age, but there is consistent evidence that estimates from models with and without such controls are very similar (see Mitnik 2017a; Mitnik 2017b).

As a measure of the long-run family income of children is not available, in all analyses, regardless of whether they pertain to short-run or long-run IGEs, I use the family income of children when they were 35-38 years old as their income measure. This is equivalent to making

$\theta_1 = 1$ ,  $W = 0$  (for all children), and  $Cov(T, W) = 0$  by construction. As a result, in the empirical analyses I am not able to assess any aspect of the GEiVE-TS model pertaining to left-side measurement error. Nevertheless, I can still draw clear conclusions regarding right-side measurement error, the key question of whether the two-sample estimation approach advanced in this paper provides upper-bound estimates of the IGE of the expectation with the instruments typically available to mobility scholars, and several other issues of interest.

### *Results*

Figures 1 to 4 allow to assess the qualitative implications of the GEiVE-TS model, and whether the two-sample estimation approach advanced here works as expected. Each figure includes two panels. The results shown in each of these panels are based on a different set of short-run measures of parental income. In the left panels, parental income pertains to when the children were 1 or 2 or 3 . . . up to 25 years old. In the right panels, the short-run measures of parental income are five-year averages centered when the children were 3 or 4 . . . up to 23 years old. The age in the horizontal axis is in all cases the average age of the parents in the sample. The descending curves in each panel show the relationship between short-run two-sample or short-run one-sample IGE estimates and parents' average age, while the ascending curve shows the relationship between estimates of the parental measurement-error slope  $\pi_1$  and that age. In Figures 1 and 2 the instruments are, respectively, the parents' and the household head's years of education. In Figure 3 the instruments are indicator variables for father's occupation, while in Figure 4 both these variables and parents' education are used to instrument parental income. In all figures, when the short-run income measures rely on five years of information, the estimates become less affected by transitory income fluctuations and the shapes of the curves become clearer.<sup>26</sup>

Tables 2, 3 and 4 present IGE estimates obtained with short-run measures of parental income and with the long-run income measure, and compare the short-run and long-run estimates. Table 2 reports the results obtained with what I will refer as the “ideal estimation strategy,” as it displays IGE estimates pertaining to the parental ages at which the measurement-error slope is equal to one. Those estimates are computed by interpolation, and therefore standard errors are not available. Table 3 shows estimates based on measures of parental income centered on the years the children were 13 years old, when the average age of the parents is close to 40. Assessing the performance of the two-sample estimation approach in this context is of eminent practical interest, as mobility scholars normally do not know the ages at which the measurement-error slopes are equal to one with their data. So, when possible with those data, they simply use estimates based on income measures taken when parents and children are close to age 40 as their best guess (a guess informed by the results obtained with other datasets). I will refer to the two-sample estimation approach implemented this way as the “feasible estimation strategy.”<sup>27</sup>

Figures 1 to 4 and Tables 2 and 3 allow to compare IGE estimates obtained with the one-sample and two-sample estimators. When conducting these comparisons, it’s important to keep two facts in mind. First, under the assumptions discussed in this paper, the probability limit of the two-sample estimator is the same as the probability limit of the one-sample estimator when the measurement-error slopes  $\pi_1$  and  $\theta_1$  are equal to one and the instruments are valid, but not necessarily at other values of those slopes or when the instruments are invalid. Second, even when the probability limits of two estimators are identical, actual estimates based on finite samples may still be different. Therefore, we should not expect estimates generated with the one- and two-sample estimators to be the same, even when they rely on the same instrument or instruments, and on the same measure of parental income. Comparing the estimates obtained

with the one-sample and two-sample estimators is nevertheless informative, and may provide a useful reference for other empirical research, e.g., for comparative research across countries that differ in the types of data they have available to estimate the IGE.

As discussed earlier, there are good reasons to expect  $\pi_1$  to increase with parental age, and to be smaller than one when the parents are younger and larger than one when the parents are older, regardless of the short-run measure of parental income employed. Figures 1 to 4 confirm these expectations. They also indicate that the measurement-error slopes are equal to one when the parents are, on average, somewhat younger than 40, and that they are somewhat larger than one at age 40 (see Tables 2 and 3 for the exact ages and slope values, respectively). In addition, and fully consistent with the GEiVE-TS model's predictions, the figures also show a clear inverse relationship between the two-sample IGE estimates and the estimates of the measurement-error slopes, with the latter increasing and the former falling at similar paces with the parents' average age. Importantly, the IGE curves are (approximately) convex to the origin. The IGE estimates are very large when the parents are younger—i.e., when they are close to 30 years old—and tend to fall very rapidly up to age 40. However, after that they tend to fall slowly or to stabilize. With each instrument (or set of instruments) and income measure used, the shape of the two-sample curve is very similar to the shape of the corresponding one-sample curve. With instruments other than parents' education, and in particular with father's occupation, the two-sample curves tend, however, to be above their one-sample counterparts.

In all figures and tables, the long-run IGE—represented in the figures by the darker-gray horizontal lines—is the IGE of the family income of children, when they were 35-38 years old, with respect to the (approximated) long-run family income of their parents. As explained earlier, for the purposes of the analyses here the former income is assumed to be the long-run income of

children. Moreover, given what we know from previous research about the children's ages at which  $\theta_1 = 1$  (Mitnik 2017a), it is likely that the 0.6 value reported as the long-run IGE estimate in the figures and tables is quite close to the estimate that would be obtained if the long-run income of children were available.

Comparing the two-sample IGE curve in each panel of each figure to the long-run IGE line makes apparent that the ideal estimation strategy works as expected: In each of the eight panels, the two-sample IGE estimate when the measurement-error slope is equal to one (a) bounds the estimate of the long-run IGE from above, and (b) provides a tighter upper bound than all estimates pertaining to ages at which the measurement-error slope is smaller than one. The first point is also apparent in Table 2, which also includes results obtained by instrumenting parental income with second-degree polynomials on each of the education variables, and with father's occupation and the household head's education together. The table also makes clear that the two-sample IGE estimates obtained when the measurement-error slopes are equal to one vary substantially across instruments, with some instruments generating much tighter upper bounds than others, and that the observed variation is very highly correlated across the two short-run measures of parental income. Using parents' education, a second-degree polynomial on parents' education, or parents' education together with father's occupation as instruments generates the tightest bounds, while instrumenting the short-run income measures with the education of the household-head (alone, or via a second-degree polynomial) provides the loosest bounds by a wide margin. The upper bounds obtained in the other two cases are in between, but are closer to those obtained with parents' education. The "ranking" of the estimates across instruments (in terms of the tightness of the bounds they supply) is somewhat different in the one sample-

context, although it is still the case that estimates based on the education of the household head provide, by far, the loosest bounds.

Crucially, the figures and Table 3 indicate that a very similar analysis applies when we focus on the results of estimating the IGE with the feasible estimation strategy. Although all two-sample estimates are smaller close to age 40 than at the ages at which the measurement-error slopes are equal to one—the “feasible estimates” are, on average across instruments and measures of parental income, about 12 percent smaller than the “ideal estimates”—the former still bound the long-run estimate from above with all instruments. (In fact, the two-sample estimates bound the long-run estimate from above at all or nearly all parental ages considered in the analysis, i.e., when the average age of the parents is between 29 and 53.) Table 4 presents the absolute and percent differences between the two-sample feasible estimates and the long-run estimate of the IGE. Depending on the instruments employed, the two-sample estimates are between 12.5 and 51.9 percent larger than the long-run estimate with the annual measure of parental income, and between 17.2 and 54.3 percent larger with the five-year measure. This wide variation in IGE estimates across instruments is not specific to the two-sample estimation of the IGE of the expectation. Indeed, Table 3 shows that the one- and two-sample estimates vary similarly across instruments (see also Table 2). Moreover, estimating the IGE of the geometric mean, in both the one-sample and the two-sample contexts, also generates a wide range of estimates.<sup>28</sup> The observed variation in estimates is a consequence of the fact that the instruments’ degree of endogeneity and the strength of their correlations with the logarithm of long-run parental income (relative to their variances) both differ across instruments.

## *Discussion*

The results of the empirical analyses indicate that the estimation approach advanced in this paper works as expected, and that the GEiVE-TS model provides a very good account of the relationship between the long-run IGE, the short-run IGE estimates generated by the two-sample GMM estimator, and the parents' ages at which their income is measured. Most crucially, the results show that both the ideal and the feasible estimation strategies work as the formal analysis leads us to expect.

The GEiVE-TS model entails that IGE estimates based on measures of parents' income obtained when they are younger should be larger than those based on measures obtained when they are older. Therefore, it is not surprising that estimates relying on annual measures of parents' income pertaining to their early 30s are larger than both the ideal and the feasible estimates, which pertain to when the parents were 37 and 40 years old, respectively. Likewise, different invalid instruments can be expected to be differentially correlated to the logarithm of long-run parental income, and to the error term of the long-run population regression function of interest. Therefore, it is not surprising that they lead to different upper-bound estimates of the IGE. Nevertheless, the magnitude of the differences revealed by the empirical analyses—across parental ages in the first case, across instruments in the second—is quite striking.

The foregoing suggests the following approach for estimating the IGE of the expectation with short-run income measures in the two-sample context. First, whenever possible, mobility scholars should use a short-run measure of parental income pertaining to when the parents were about 40 years old. If this is not possible, measures obtained when parents were somewhat older should be strongly preferred to measures obtained when parents were somewhat younger. This is so because the empirical results suggest that the IGE estimates become much less informative in

the latter case, while estimates based on measures pertaining to somewhat older ages are likely to still bound the long-run IGE from above.

Second, mobility scholars should estimate the IGE using a variety of instruments, and then select the estimate that provides the tightest bound as the preferred estimate. Under this approach, searching for “best invalid instruments”—e.g., looking for additional instruments beyond those typically employed, using multiple instruments simultaneously, and exploring the effects of alternative functional forms—may have a large payoff.

## **Conclusion**

The IGE conventionally estimated in the mobility literature pertains to the conditional geometric mean of children’s income, which is at odds with all the interpretations imposed on its estimates. In addition, the conventional IGE makes studying gender and marriage dynamics in intergenerational processes a very difficult enterprise, and leads to IGE estimates affected by (potentially severe) selection biases. For these reasons, Mitnik and Grusky (2017a) have called for replacing that IGE by the IGE of the expectation. As in many countries the only information available for the estimation of intergenerational elasticities is short-run income variables drawn from two independent samples, making the IGE of the expectation the workhorse intergenerational elasticity—as Mitnik and Grusky propose—requires that the methodological knowledge necessary to estimate it with this information is made available.

In this paper, I have made three contributions to this goal. I have put forth a two-sample GMM estimator of the exponential regression model, the GMM-E-TS estimator, which can be employed to estimate the IGE of the expectation. The estimator may be used both with data obtained through simple random sampling and with data that are the result of a complex sampling design. Moreover, because the GMM-E-TS estimator is a two-equation rather than a



two-step estimator, the computation of standard errors only involves standard formulas. Therefore, the estimator may be used widely and easily, and is very well suited for playing for the estimation of the IGE of the expectation the role that the TSTOLS estimator has played for the estimation of the conventional IGE.

I also have advanced a generalized error-in-variables model for the estimation of the IGE of the expectation with the GMM-E-TS estimator and short-run income measures. This model provides an account of the relationship between the long-run IGE, the ages at which children's and parents' incomes are measured, and the probability limit of the GMM-E-TS estimator with short-run income variables and with both valid and invalid instruments. A central implication of the model is that when the measurement-error slopes are equal to one, estimation of the IGE of the expectation with that estimator and the instruments typically available can be expected to provide upper-bound estimates of the long-run IGE.

Lastly, I have used PSID data to assess the performance of the estimation approach developed in the paper. The empirical results indicate that the estimator works as expected, and that the observed lifecycle and amplification biases are in close agreement with the qualitative predictions of the formal analysis. They also provide support for the hypothesis that measurement-error slopes are equal to one close to age 40, and show that although estimates generated with the invalid instruments typically available to mobility scholars all bound the long-run IGE from above, they vary substantially across instruments. This suggests that, in addition to using short-run income measures obtained as close to age 40 as possible—so as to minimize lifecycle biases—mobility scholars should estimate the IGE with a variety of instruments, and then select the estimate that provides the tightest upper bound as the preferred estimate.

## Appendix

In this appendix I provide some additional details on the sample and variables used in the empirical analyses.

To select the sample, and similarly to Hertz (2007), I define “child” broadly to include anyone of the right age reported in the PSID to be either the son, daughter, stepson, stepdaughter, nephew, niece, grandson or granddaughter of the household head or his wife (or long-term partner).<sup>29</sup> As Hertz (2007:35) put it, “the idea is to look at the relation between children’s income and the income of the households in which they were raised, even if that household was not, or not always, headed by their mother or father.” Similarly, when the children are 1-17 years old, the “father” is the household head (if the head is male), while the “mother” is either the household head (if the head is female) or the head’s wife or long-term partner. When the children are older than 17, the father and mother are those determined to be the father and mother at age 17.

When estimating the IGE of the expectation, the dependent variable is the average family income of children when they were 35-38 years old. I only estimate the IGE of the children’s expected family income, not of their expected individual earnings. The reason is that the IGE of children’s individual earnings needs to be estimated separately by gender, but the available PSID sample is rather small, even when men and women are pooled (as I do in all analyses).

For both parents and children, the annual measures of family income are based on the PSID notion of “total family income.” But as the income components the PSID used to compute total family income are effectively affected by top coding in the period 1970-1978 (i.e., top codes were not only in place but were “binding” in that period for some people), and the PSID-computed family income for those years is based on these top-coded values, I proceeded as

follows: (a) I addressed the top-coding of all income components in 1970-1978 by using Pareto imputation (Fichtenbaum and Shahidi 1988), and (b) I recomputed total family income for those years with the Pareto-imputed component variables.

I use as instruments the parents' and the household head's years of education when the children were 13 years old (rather than some other age) because (a) when the children are 13 years old the average age of the parents is close to 40 years old, and (b) I am particularly interested in examining estimates obtained with parents' information pertaining to when they were close to 40 years old. I use the household head's years of education but not the father's years of education, which is the instrument most often used in the mobility literature. I do not use this instrument because that would require dropping from the sample those children who grew up without a father, which is likely to generate selection bias. Nevertheless, if the father is present in the household, in the vast majority of cases he is coded as household head by the PSID. Therefore, the household head's years of education is similar, but not identical, to the father's years of education. I do use father's occupation as instrument. This is not a problem because this variable is categorical; children that did not provide information on their fathers' occupation (regardless of the reason) can be coded in a separate category, and this is what I do (see Table 1).

## Notes

<sup>1</sup> For detailed discussions of these generalized error-in-variables models, see Mitnik (2017a:7-12; 2017b:8-10) and Nybom and Stuhler (2016).

<sup>2</sup> Using the PPML estimator does not require assuming that the dependent variable has a conditional Poisson (or any other particular) distribution. PML estimators are semi-parametric; they are consistent regardless of the actual distribution of dependent variable, provided that the mean function is correctly specified (Gourieroux, Monfort, and Trognon 1984).

<sup>3</sup> Samples of child-parents pairs in which lifetime income is observed for both generations have not been available in any country.

<sup>4</sup> Jerrim et al. (2016) point out that this has proven to be the only feasible approach in Australia, China, France, Japan, Italy, South Africa, Spain and Switzerland. Other countries in the same situation are Argentina (Jiménez and Jiménez 2009), Brazil (Dunn 2007), Chile (Núñez and Miranda 2011), and Ecuador, Nepal, Peru and Singapore (Grawe 2004). In a few cases a closely related estimator, the two-sample instrumental-variable (TSIV) estimator, has been used instead of the TSTSLS estimator (see Inoue and Solon 2010).

<sup>5</sup> The parameter  $\beta_1$  is (also) the IGE of the expectation only when the error term satisfies very special conditions (Santos Silva and Tenreiro 2006; Petersen 2017; Wooldridge 2002:17).

<sup>6</sup> Importantly, the methodological problems discussed in this paragraph can't be solved by replacing zeros by "small values" (Mitnik and Grusky, 2017:Section III.C).

<sup>7</sup> This claim assumes that the model in the first step of Mullahy's estimator is a linear model, which is not strictly necessary (see Mullahy 1997:Ftn. 20).

<sup>8</sup> This doesn't involve any loss of generality because it can always be achieved by simply changing the monetary units used to measure income, i.e., by dividing the children's income

variable by its mean, and the parental income variable by the exponential of the mean of its logarithmic values minus one.

<sup>9</sup>  $E(\Psi|t) = 0$  follows from  $E(\Psi|x) = 0$  (see Equation [4]) and assumption A4.

<sup>10</sup> Assumption A2 pertains to any  $c > 0$ , and not just to  $\alpha_1$ , in order for this identification condition to be feasible. (Arguably,  $c \in (0, k)$ , with  $k = 1$  or perhaps  $k = 2$  should suffice in the context at hand, as we generally expect the IGE of the expectation to be in the unit interval; conversely, in a fully-general context A2 would need to be formulated in terms of  $c \in \mathbb{R}$ .)

<sup>11</sup> The estimator also is approximately consistent if, for all  $t$ ,  $Var(R|t)$  is “very small” (regardless of whether it is constant or not across values of  $T$ ). This is essentially the assumption that Carroll et al. (2006) made to derive their one-sample IV estimator.

<sup>12</sup> Equations [9], [9a] and [9b] assume  $E(Y) = 1$ , which is true by hypothesis, and  $E(E(\ln X | T)) = 1$ . The latter follows from  $E(\ln X) = 1$ , which is true by hypothesis.

<sup>13</sup> Equation [10] assumes that  $E(Y) = 1$  and that  $E(E(\ln X | T)) = 1$ . This follows immediately from  $E(Y) = 1$  and  $E(\ln X) = 1$ , which are true by hypothesis. In the last step I used that  $Cov(\Psi, T) = Cov(E(\Psi|T), T)$ . Indeed, applying the law of total covariance, we may write  $Cov(\Psi, T) = E(Cov(\Psi, T)|T) + Cov(E(\Psi|T), E(T|T)) = Cov(E(\Psi|T), T)$ .

<sup>14</sup> In the case of the conventional IGE, the fact that the standard conclusion relies on (counterfactually) assuming that the long-run income variables are available was noted by Jerrim et al. (2016).

<sup>15</sup> See note 8.

<sup>16</sup> This assumes that  $E(Z) = 1$  and  $E(E(\ln S_k | D)) = 1$ . The former is true by hypothesis while the latter follows from  $E(\ln S_k) = 1$ , which is true by hypothesis.

<sup>17</sup> Importantly, as the left-side measurement error pertains to the income variable rather than its

logarithm, the GEiVE-TS model does not preclude that the observed income variable is equal to zero, and therefore children without any short-run income (e.g., children who are unemployed the year in which their incomes are measured) are included in all analyses.

<sup>18</sup> Note that as here  $\tilde{\gamma}_1 = \gamma_1$ , if an instrument is valid (invalid) in the long-run context it is also valid (invalid) in the short-run context, and vice-versa.

<sup>19</sup> As I did, but in that case only to facilitate the presentation, in my derivation of those equations above.

<sup>20</sup> This is possible because the PPML estimator used in the second step is both an M-estimator and a GMM estimator of the exponential regression model.

<sup>21</sup> It is not possible to select a PSID sample that (a) has the minimum size required by those analyses, and (b) can be used to construct long-run income measures for parents and children simultaneously.

<sup>22</sup> A standard assumption in two-sample estimation is that the main and auxiliary samples have been randomly drawn from the same population (e.g., Jerrim et al. 2016). Typically, this assumption holds rather imperfectly in the contexts in which two-sample estimation of IGEs is actually carried out. Nevertheless, it is very helpful that it holds perfectly here. Indeed, if this were not the case the empirical results would reflect not only the performance of the GMM-E-TS estimator and the GEiVE-TS model, but also the differences between the populations from which the samples were drawn. Relatedly, even if the common-population assumption is satisfied, sampling variability could also lead to misleading results if any of the samples is small.

<sup>23</sup> I use the statistical package Stata to produce all estimates. See Author (2017) for details on the official and user-written commands that implement the estimators just mentioned, including the GMM-E-TS estimator introduced in this article. In an auxiliary analysis, whose results I report in

note 28, I also use the OLS estimator, the TSLS estimator, and a GMM version of the TSTSLS estimator to estimate the IGE of the geometric mean. In all cases in which I employ a GMM estimator, I use the iterative version of the estimator (with a maximum of eight iterations), as this likely improves efficiency in finite samples (see Hall 2005:Sec. 2.4 and 3.6).

<sup>24</sup> In the Appendix I explain why I use parents' education and the household head's education as instruments but not father's education, which is the typical approach in the literature. I also explain why I focus on the parents' and the household head' education when the children were 13 years old.

<sup>25</sup> Cluster-robust, and not just robust, standard errors are needed because of the way the samples were generated (see the previous paragraph).

<sup>26</sup> This is the main motivation for including five-year parental income measures—which are not likely to be available in the contexts in which two-sample estimation is carried out—in the analyses conducted here: It helps compensate, at least partially, for the small size of the samples.

<sup>27</sup> The tables include results obtained with all instruments mentioned earlier. However, I have not included in the paper figures in which the estimates rely on instrumenting short-run parental income with second-degree polynomials on the education variables, or with both the father's occupation and the household head's education, as they add very little information to that provided by the other figures.

<sup>28</sup> With the same sample, instruments and income measures employed to produce the results shown in Tables 3 and 4 (i.e., those pertaining to when the parents' average age is close to 40), the percent differences between two-sample estimates of the IGE of the geometric mean of children's income and the corresponding long-run estimate range, depending on the instruments used, from -1.9 to 33.5 percent with the annual measure of income, and from -0.6 to 39.3 percent

with the five-year measure. Similarly, the percent differences in one-sample estimates range from 8.3 to 33.5 percent (annual measure), and from 10.4 to 39.3 percent (five-year measure). Using a larger set of instruments, Jerrim et al. (2016) report a much larger range of variation for the two-sample estimates of the IGE of the geometric mean of men's earnings with respect to father's earnings.

<sup>29</sup> By convention, the PSID codes a man as household head and his spouse as wife, i.e., it does not code a woman as household head and his spouse as husband.



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**Table 1: Descriptive Statistics (unweighted values)**

Child's gender (% female)	51.8	Father's occupation (%)			
Child's age		Management occs.	10.2	Construction trades	10.2
Mean	36.6	Business operations specialists	0.6	Extraction workers	0.4
Standard deviation	0.6	Financial specialists	1.8	Installation, maintenance, and repair workers	7.5
Child's family income		Computer and mathematical occs.	1.1	Production occupations	12.8
Mean	91,625	Architecture and engineering occs.	3.3	Transportation and material moving occs.	10.2
Standard deviation	88,398	Life, physical, and social science occs.	0.9	Military specific occupations	2.5
Average parental age		Community and social services occs.	0.9	Doesn't know, refused	11.6
Mean	40.1	Legal occupations	0.4	Not applicable	1.5
Standard deviation	6.7	Education, training, and library occs.	2.7		
Average parental income		Arts, design, entert., sports, and media occs.	1.6		
Mean	78,858	Healthcare practitioners and technical occs.	1.6		
Standard deviation	64,185	Protective service occupations	2.8		
Parents' years of education at child age 13		Food preparation and serving occupations	0.5		
Mean	21.9	Building and grounds clean. and maint. occs.	1.5		
Standard deviation	6.9	Personal care and service occs.	0.4		
Household head' years of education at child age 13		Sales occupations	8.6		
Mean	12.3	Office and administrative support occs.	3.0		
Standard deviation	2.9	Farming, fishing, and forestry occs.	1.9		
N	827				

Note: Monetary values are in 2012 dollars (adjusted by inflation using the Consumer Price Index for Urban Consumers - Research Series). The average parental age and income pertain to when the children were 1-25 years old. Father's occupation is coded as "not applicable" when there was no father/surrogate, the father was deceased, or the father never worked. To save space only the long-run parental income variables are included in the table. The empirical analyses also employ many short-run parental income variables.

**Table 2: Two-sample short-run estimates of the IGE of expected income when measurement-error slope is equal to one, compared to one-sample short-run estimates when measurement-error slope is equal to one, and to long-run estimate**

	Short-run parental income				Long-run parental income
	One year of information		Five years of information		
	Two samples	One sample	Two samples	One sample	
IGE of children's expected income					
Short-run estimates, with short-run income instrumented by:					
Parents' education	<b>0.80</b>	<b>0.82</b>	<b>0.77</b>	<b>0.79</b>	
Parents' education and its square	<b>0.79</b>	<b>0.83</b>	<b>0.78</b>	<b>0.80</b>	
Household head's education	<b>0.99</b>	<b>0.93</b>	<b>0.97</b>	<b>0.92</b>	
Household head's education and its square	<b>1.05</b>	<b>1.03</b>	<b>1.01</b>	<b>0.99</b>	
Father's occupation	<b>0.83</b>	<b>0.69</b>	<b>0.83</b>	<b>0.73</b>	
Parents' education and father's occupation	<b>0.80</b>	<b>0.72</b>	<b>0.77</b>	<b>0.74</b>	
Householdhead's education and father's occupation	<b>0.87</b>	<b>0.74</b>	<b>0.85</b>	<b>0.78</b>	
Long-run estimate					<b>0.60</b> (0.073)
Average parental age		<b>37.0</b>	<b>37.8</b>		<b>40.5</b>

Note: Short-run estimates were computed by interpolation. Point estimates are in bold, standard error (for the long-run estimate only) is in parentheses. The long-run estimate was generated with the PPML estimator. The short-run one- and two-sample estimates underlying the short-run interpolated figures shown in the table were generated with the GMM-IVP and the TS-GMM-E estimators, respectively

**Table 3: Two-sample short-run estimates of the IGE of expected income when average parental age is close to 40, compared to one-sample short-run estimates when average parental age is close to 40, and to long-run estimate**

	Short-run parental income				Long-run parental income
	One year of information		Five years of information		
	Two samples	One sample	Two samples	One sample	
IGE of children's expected income					
Short-run estimates, with short-run income instrumented by:					
Parents' education	<b>0.69</b> (0.079)	<b>0.72</b> (0.086)	<b>0.71</b> (0.079)	<b>0.73</b> (0.083)	
Parents' education and its square	<b>0.69</b> (0.077)	<b>0.72</b> (0.083)	<b>0.72</b> (0.077)	<b>0.73</b> (0.080)	
Household head's education	<b>0.86</b> (0.110)	<b>0.85</b> (0.112)	<b>0.88</b> (0.106)	<b>0.85</b> (0.099)	
Household head's education and its square	<b>0.91</b> (0.102)	<b>0.92</b> (0.110)	<b>0.92</b> (0.099)	<b>0.91</b> (0.095)	
Father's occupation	<b>0.70</b> (0.085)	<b>0.69</b> (0.078)	<b>0.73</b> (0.090)	<b>0.69</b> (0.082)	
Parents' education and father's occupation	<b>0.67</b> (0.065)	<b>0.69</b> (0.065)	<b>0.70</b> (0.067)	<b>0.69</b> (0.066)	
Householdhead's education and father's occupation	<b>0.75</b> (0.074)	<b>0.72</b> (0.068)	<b>0.78</b> (0.076)	<b>0.72</b> (0.069)	
Long-run estimate					<b>0.60</b> (0.073)
Measurement-error slope ( $\pi_1$ )		<b>1.10</b>	<b>1.07</b>		

Note: Point estimates are in bold, standard errors are in parentheses. The long-run estimate was generated with the PPML estimator. The short-run one- and two-sample estimates were generated with the GMM-IVP and the TS-GMM-E estimators, respectively.

**Table 4: Absolute and percent differences between two-sample short-run estimates when parental age is close to 40 and the long-run estimate of the IGE of expected income**

	Parental information			
	One year		Five years	
	Diff	% diff.	Diff	% diff.
Estimates with short-run income instrumented by:				
Parents' education	0.09	15.6	0.11	18.8
Parents' education and its square	0.09	15.5	0.12	20.1
Household head's education	0.26	43.6	0.28	46.8
Household head's education and its square	0.31	51.9	0.32	54.3
Father's occupation	0.10	17.5	0.13	22.6
Parents' education and father's occupation	0.07	12.5	0.10	17.2
Household head's education and father's occupation	0.15	25.5	0.18	30.8

Note: The differences indicate how much larger the two-sample short-run IGE estimates are compared to the long-run estimate. The former were generated with the TS-GMM-E estimator. The latter was generated with the PPML estimator.

Figure 1: Two-sample estimation of the IGE of children's expected income as a function of parental age  
Instrument: Parents' education

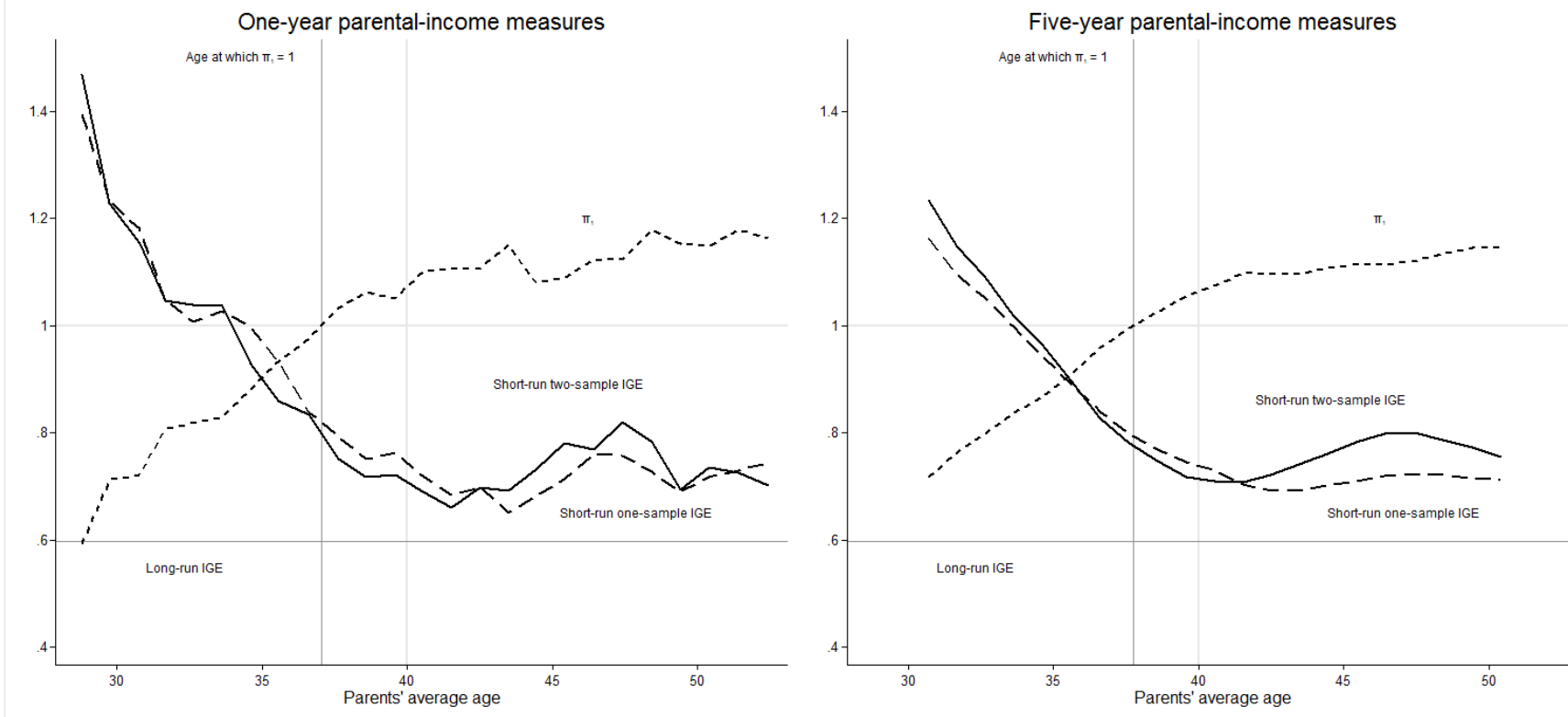




Figure 2: Two-sample estimation of the IGE of children's expected income as a function of parental age  
 Instrument: Household head's education

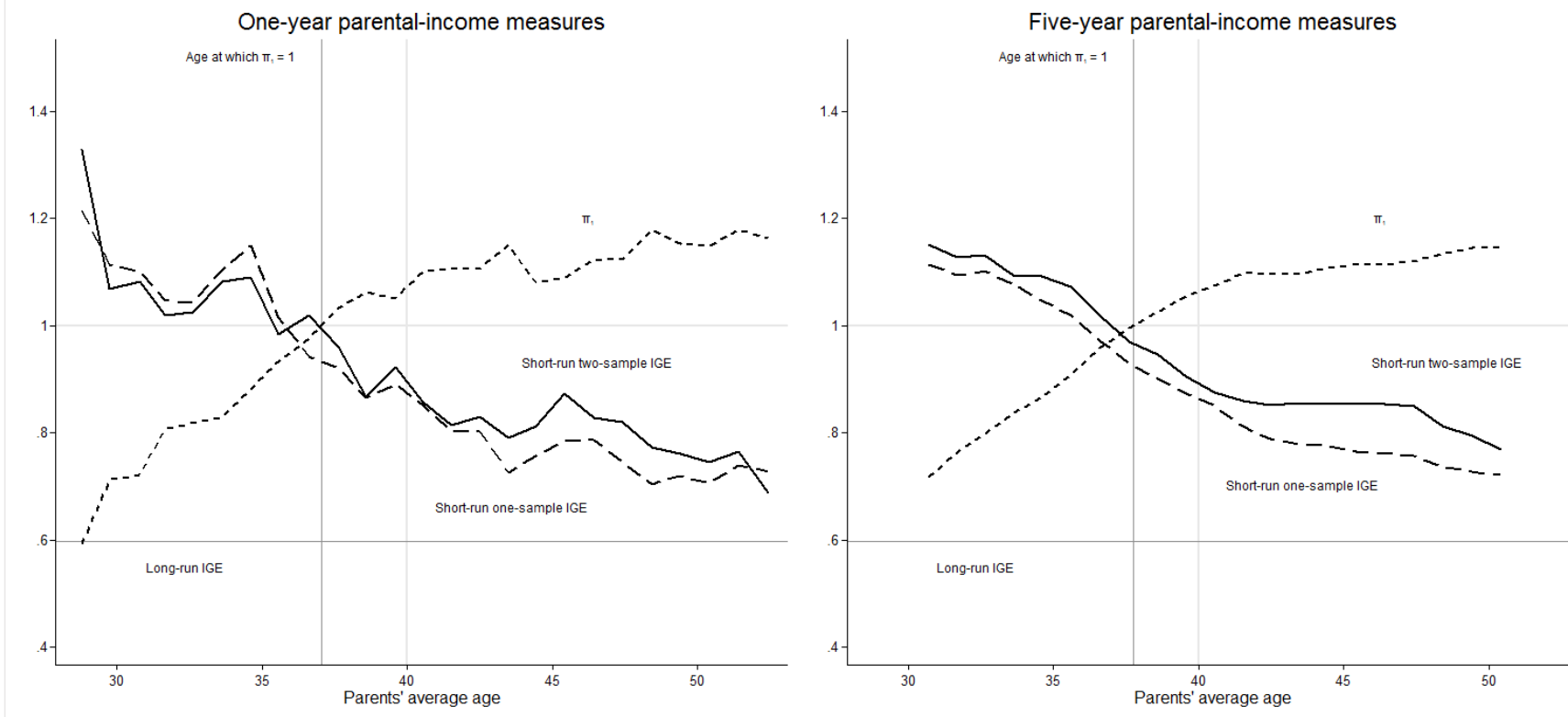


Figure 3: Two-sample estimation of the IGE of children's expected income as a function of parental age  
Instrument: Father's occupation

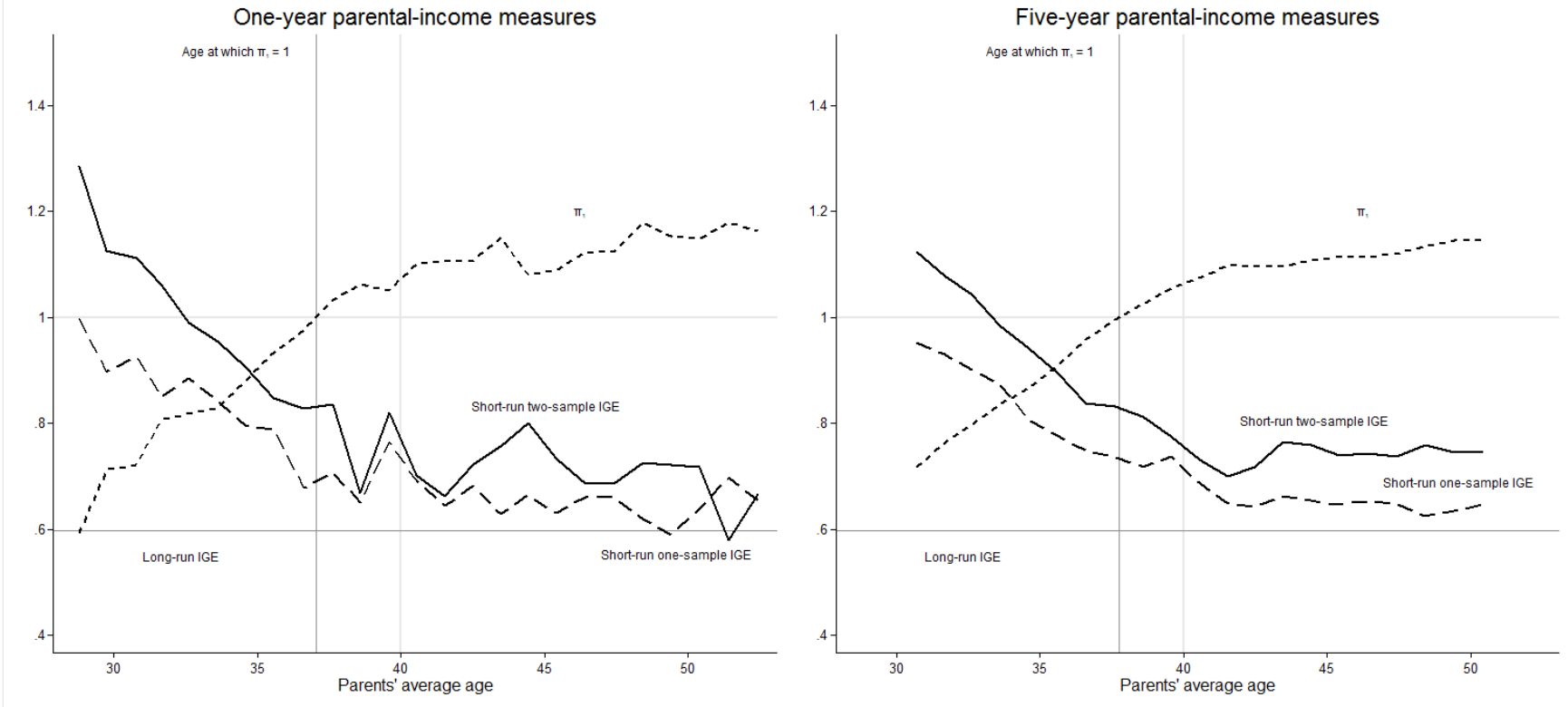


Figure 4: Two-sample estimation of the IGE of children's expected income as a function of parental age  
 Instruments: Parents' education and father's occupation

