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ESTIMATING THE INTERGENERATIONAL ELASTICITY OF EXPECTED INCOME WITH SHORT-RUN INCOME MEASURES: A GENERALIZED ERROR-IN-VARIABLES MODEL

Pablo A. Mitnik
(pmitnik@stanford.edu)

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Abstract

It has recently been argued that the intergenerational income elasticity (IGE) ubiquitously estimated in the economic mobility literature should be replaced by the IGE of expected income. For this to be possible, a generalized error-in-variables model for the estimation of the latter IGE with short-run income measures is required. This paper derives an approximate closed-form expression for the probability limit of the Poisson Pseudo Maximum Likelihood (PPML) estimator and uses it to develop the needed error-in-variables model. It also evaluates the model with data from the Panel Study of Income Dynamics. The results of the empirical analyses offer clear support for the account of lifecycle and attenuation biases provided by the model, and show that the strategy most commonly employed to estimate the conventional IGE with short-run income variables can also be used for the estimation of the IGE of expected income with the PPML estimator.

I. Introduction

The intergenerational elasticity (IGE) is the workhorse measure of intergenerational economic mobility. It has been widely estimated over the last four decades, very often with the goal of providing a summary assessment of the level of income or earnings mobility within a country (for reviews, see Solon 1999:1778-1788; Corak 2006; Mitnik et al. 2015:7-15). Beyond this elementary goal, the IGE has been used, among other purposes, to conduct comparative analyses of economic mobility and persistence across countries, regions, demographic groups, cohorts, and time periods (e.g., Chadwick and Solon 2002; Hertz 2005, 2007; Aaronson and Mazumder 2008; Björklund and Jäntti 2000; Meyer and Lopoo 2008; Bloome and Western 2011); to examine the relationship between cross-sectional economic inequality and mobility across generations (e.g., Corak 2013; Bloome 2016); and to theoretically model and empirically study the impact of social policies and political institutions on inequality of opportunity (e.g., Solon 2004; Bratsberg et al. 2007; Ichino et al. 2011; Landersø and Heckman 2016).

In spite of the very central place that the IGE holds in the study of intergenerational mobility, Mitnik and Grusky (2017) have recently shown that this elasticity has been specified in a way that is inconsistent with the various interpretations imposed on its estimates. While the IGE has been widely construed as pertaining to the expectation of children's income conditional on their parents' income—as apparent, for instance, in its archetypical interpretation as a measure of regression to the (arithmetic) mean—in fact it pertains to the conditional geometric mean of the children's income. This not only invalidates all conventional interpretations of the IGE, but also generates serious methodological problems. At their root, these are the result of a very simple fact, i.e., that the geometric mean is undefined for variables including zero in their support. This fact markedly hampers the study of gender and marriage dynamics in intergenerational processes, as it (a) makes it impossible to determine the extent to which parental economic advantage is transmitted through the labor market among women, and (b) greatly hinders research on the role that marriage plays in generating the observed levels of intergenerational persistence in family income. Equally important, estimation of the IGE with the annual or other short-run proxy income measures that are available is almost certainly affected by substantial selection biases, which result from mobility scholars' expedient of dropping children with zero income from samples (to address what is perceived as the problem of the logarithm of zero being undefined). This makes the (currently widespread) use of the IGE of men's individual earnings as an index of economic persistence and mobility in a country a rather problematic practice.

Mitnik and Grusky (2017) have argued that these conceptual and methodological problems can be solved all at once by simply replacing the IGE of the geometric mean—the de facto estimated IGE—by the IGE of the expectation—the IGE that mobility scholars thought they were estimating—as the workhorse intergenerational elasticity. They have also called for effectuating such replacement. Their call, however, confronts a key methodological obstacle.

Over the last 40 years or so, the mobility field has been centrally concerned with the lifecycle and attenuation biases that may result from the estimation of the conventional IGE with short-run proxy income measures, and the methodological strategies that may be used to avoid them. As a result, a large body of knowledge on these matters has accumulated over this period. A central achievement in this regard is Haider and Solon's (2006) generalized error-in-variables (GEiV) model, which supplies the current methodological justification for the estimation of the conventional IGE with proxy variables that satisfy some conditions (see Nybom and Stuhler 2016 for an illuminating discussion of this model). In stark contrast, the IGE of the expectation was estimated for the first time by Mitnik et al. (2015), and there is very little methodological knowledge regarding its estimation with short-run measures. In particular, there is nothing comparable to the GEiV model that would apply to the IGE of the expectation.

In this paper I tackle head on this key methodological obstacle to redefining the IGE as Mitnik and Grusky (2017) advocate. The task is made difficult by the lack of closed-form expressions for the probability limits of the estimators of the IGE of the expectation, and by the fact that in nonlinear models the bias induced by right-side measurement error may be in any direction. I deal with these difficulties by deriving an approximated closed-form expression for the probability limit of the Poisson Pseudo Maximum Likelihood (PPML) estimator based on Taylor-series expansions, and by relying on substantive knowledge to sign some key quantities in conducting proofs. This allows me to advance a formal measurement-error model, the GEiVE model, which plays for the IGE of the expectation the same role that the GEiV model plays for the conventional IGE.

When datasets with the necessary information are available, a measurement-error model may be empirically evaluated. Here I use U.S. samples from the Panel Study of Income Dynamics (PSID) for this purpose. My results indicate that, when estimating the IGE of the expectation with proxy measures, the observed lifecycle and attenuation biases are in close agreement with the GEiVE model's predictions. Equally important, the results provide evidence on the patterns and magnitudes of lifecycle and attenuation biases that are relevant for the specification of estimation strategies and the interpretation of results obtained with other datasets.

Similarly to what the GEiV model and the associated evidence do for the estimation of the conventional IGE, the GEiVE model and the evidence I present here supply a methodological justification for the estimation of the IGE of the expectation with proxy income variables that satisfy some conditions—that is, with income measures obtained when children and parents are close to 40 years old and, in the case of parents, based on several years of information. This eliminates the key obstacle to replacing the conventionally estimated IGE by the IGE of the expectation, as advocated by Mitnik and Grusky (2017).

The structure of the rest of the paper is as follows. I start by explaining why the conventional IGE has been misinterpreted, the methodological problems that the estimation of that IGE generates, and Mitnik and Grusky's (2017) proposal to redefine the IGE used as the workhorse measure of economic mobility. Next, I summarize the GEiV model, introduce the new GEiVE model, and present the empirical analyses. The last section distills the main conclusions of the paper. Mathematical proofs and supplementary materials, including additional empirical results, are collected in an Online Appendix.

II. The IGE of what? Redefining the workhorse intergenerational elasticity

As already indicated, the conventionally estimated IGE has been widely misinterpreted. While mobility scholars have interpreted it as the elasticity of the expectation of children's income or earnings conditional on parental income, that IGE pertains in fact to the conditional geometric mean. Closely following Mitnik and Grusky's (2017) analysis, the standard population regression function (PRF) posited in the literature, which assumes the elasticity is constant across levels of parental income, is:

$$E(\ln Y\,|x) = \beta_0 + \beta_1 \ln x, \qquad [1]$$

where Y is the child's long-run income or earnings, X is long-run parental income or father's earnings, β_1 is the IGE as specified in the literature, and I use expressions like "Z$|w$" as a shorthand for "$Z|W = w$." The parameter β_1 is not, in the general case, the elasticity of the conditional expectation of the child's income. This would hold as a general result only if $E(\ln Y|x) = \ln E(Y|x)$. But, due to Jensen's inequality, the latter is not the case. Instead, as $E(\ln Y\,|x) = \ln \exp E(\ln Y\,|x)$, and $GM(Y|x) = \exp E(\ln Y\,|x)$, Equation [1] is equivalent to

$$\ln GM(Y|x) = \beta_0 + \beta_1 \ln x, \qquad [2]$$

where GM denotes the geometric mean operator. Therefore, β_1 is the elasticity of the conditional geometric mean, i.e., the percentage differential in the geometric mean of children's long-run income with respect to a marginal percentage differential in parental long-run income.[1]

As the geometric mean is undefined whenever an income distribution includes zero in its support, the IGE is undefined as well when this is the case. Mitnik and Grusky (2017: Section IV) have shown that this has very negative consequences for the study of gender and marriage dynamics in intergenerational processes. First, because a sizable share of women never join the (paid) labor force, the "zeros problem" makes impossible to determine the extent to which parental economic advantage—as measured by the family-income IGE—is transmitted through the labor market among women, and whether there are gender differences in this regard (as the relevant estimand, the women's earnings IGE, is undefined). Second, the zeros problem greatly hinders research on the role that marriage plays in generating the observed levels of intergenerational persistence in family income. This is so because a measure that is central for this research—the IGE of the income contributed by a spouse with respect to a child's own parental income—is also undefined as long as there are children that are single and spouses of married children that contribute no income.

[1] The parameter β_1 is (also) the IGE of the expectation only when the error term satisfies very special conditions (Santos Silva and Tenreyro 2006; Petersen 2017; Wooldridge 2002:17).

The unwitting reliance of the mobility field on the IGE of the geometric mean also has very negative consequences for the widespread practice of using the IGE of men's individual earnings and—less frequently—of men's and women's family income as indices of economic persistence and mobility in a country. Indeed, the IGE is defined in terms of long-run earnings and family income, and it may be reasonable to assume, as an approximation, that these long-run measures are positive rather than nonnegative.[2] But estimation is nevertheless adversely affected by the ubiquitous need to use short-run measures of children's economic status (e.g., sons' earnings in a particular year) as proxies for their unavailable long-run counterparts (e.g., sons' average lifetime earnings). This is the case because the short-run measures almost always have a substantial probability mass at zero even when the corresponding long-run measures don't.[3] Assuming that zero is not in the support of the annual or other short-run children's measures employed for estimation is never a reasonable approximation in the case of men's earnings (due to unemployment and other forms of nonemployment), and it is not a reasonable approximation in most countries and times—the potential exception being countries with a highly developed

[2] As conditions like severe physical disability, severe mental illness, and imprisonment affect some children over their entire adult lives, strictly speaking zero is in the support of all measures of children's long-run income, including men's earnings and men's and women's family income.

[3] This is less of a problem for short-run parental measures, as (a) they typically are multiyear averages (more on this later), (b) parents with zero income for extended periods of time are unlikely to be able to raise their kids themselves, and (c) there are good reasons to use as parental income the income of the family in which the child lived when growing up (see, e.g., Hertz 2007:35).

welfare state—in the case of family income.[4] As argued in detail by Mitnik and Grusky (2017: Section III), dropping children without earnings or income from samples (the near-universal expedient employed to address this "short-run zeros problem") generates substantial selection biases, and all approaches that might be used to try to avoid those biases, including imputing "small values" to children with zero earnings or income, are very unattractive due to a combination of methodological and pragmatic reasons.

To address these problems, and similarly to what Santos Silva and Tenreyro (2006) did in the field of international trade, Mitnik and Grusky's (2017) have called for redefining the workhorse measure of economic mobility. This entails replacing the PRF of Equation [1] by a PRF whose estimation delivers estimates of the IGE of the expectation in the general case. Under the assumption of constant elasticity, that PRF can be written as:

$$\ln E(Y|x) = \alpha_0 + \alpha_1 \ln x, \qquad [3]$$

where $Y \geq 0, X > 0$ and $\alpha_1 = \frac{d \ln E(Y|x)}{d \ln x}$ is the percentage differential in the expectation of children's long-run income with respect to a marginal percentage differential in parental long-run income. Crucially, (a) all interpretations incorrectly applied to the conventional IGE are correct or approximately correct under this formulation (see Mitnik and Grusky 2017: Section V.A), and (b) the IGE of the expectation is fully immune to the methodological problems affecting the IGE of the geometric mean and, in particular, is very well suited for studying the role of marriage in the intergenerational transmission of advantage (see Mitnik and Grusky 2017: Section V.B for details; and Mitnik et al. 2015:64-68 for an empirical application).

[4] For the prevalence of families with zero annual income in the U.S. see, e.g., Chetty et al. (2014: Online Appendix Table IV).

III. The GEiV model

When actual measures of long-run income or earnings for parents and children are available, Equation [1] may be consistently estimated by Ordinary Least Squares (OLS). However, such measures are almost never available, only short-run proxy variables are. Estimating Equation [1] by OLS after substituting the latter variables for the former opens the door to the two types of biases widely discussed in the literature. First, as income- and earnings-age profiles differ across economic origins, lifecycle biases result from estimating the conventional IGE with proxy measures taken when parents or children are too young or too old to represent lifetime differences well (e.g., Black and Devereux 2011). Second, in the case of the parental variables, the combination of transitory fluctuations and true measurement error produces substantial attenuation bias (see, e.g., Solon 1999; Mazumder 2005). The joint analysis of these biases is provided by the GEiV model (Haider and Solon 2006).

For the sake of simplicity, I assume that all children are from the same cohort and, following Haider and Solon (2006), present separate analyses of measurement error in the left- and right-side variables of Equation [1]. We may write the GEiV model's assumptions regarding left-side measurement error as:

$$\ln Z_t = \lambda_{0t} + \lambda_{1t} \ln Y + V_t \qquad [4]$$

$$E(V_t) = 0 \qquad [5]$$

$$Cov(V_t, \ln Y) = 0 \qquad [6]$$

$$Cov(V_t, \ln X) = 0, \qquad [7]$$

where $Z_t > 0$ is children's income or earnings at age *t*, $Y > 0$, $\lambda_{0t} + V_t$ is the (additive) measurement error in the logarithm of the short-run measure as a proxy for the logarithm of the long-run measure if $\lambda_{1t} = 1$, and λ_{1t} captures left-hand lifecycle bias and thus may be different from one and varies with t. Moving now to right-side measurement error, the GEiV model's assumptions are:

$$\ln S_k = \eta_{0k} + \eta_{1k} \ln X + Q_k \quad [8]$$

$$E(Q_k) = 0 \quad [9]$$

$$Cov(Q_k, \ln X) = 0 \quad [10]$$

$$Cov(Q_k, \ln Y) = 0, \quad [11]$$

where $S_k > 0$ is parents' income or earnings around age k, $X > 0$, $\eta_{0k} + Q_k$ is the (additive) measurement error in the logarithm of the short-run measure as a proxy for the logarithm of the long-run measure if $\eta_{1k} = 1$, and η_{1k} captures right-hand lifecycle bias and thus may be different from one and varies with parents' age. Crucially, Equations [4] to [6] and [8] to [10] do not involve empirically refutable claims; rather, they are true by construction, as they simply define linear projections of $\ln Z_t$ and $\ln S_t$ on $\ln Y$ and $\ln X$, respectively (the very weak regularity conditions underlying linear projections certainly obtain in this context). In contrast, Equations [7] and [11] are empirical assumptions, which are not expected to hold perfectly but to constitute good approximations.[5]

If instead of estimating the PRF of Equation [1]—and leaving implicit the subscripts t and k from now on to simplify notation—one estimates by OLS a PRF in which $\ln Z$ is substituted for $\ln Y$, the probability limit of the slope coefficient is:

[5] Although not always stated explicitly, the GEiV model also assumes that $Cov(V_t, Q_k) = 0$, at least for any t and k for which $\lambda_{1t} \cong \eta_{1k} \cong 1$ (see, e.g., Nybom and Stuhler 2016: Eq. 2). This is also an empirical assumption. I ignore it in what follows, as it plays no role when the analyses of measurement error in the left- and right-side variables are conducted separately.

$$\tilde{\beta}_1 = \frac{Cov(\ln Z\,, \ln X)}{Var(\ln X)} = \lambda_1 \beta_1 + \frac{Cov(V, \ln X)}{Var(\ln X)}. \qquad [12]$$

Likewise, the probability limit of the slope coefficient, when $\ln S$ is substituted for $\ln X$, is:

$$\check{\beta}_1 = \frac{Cov(\ln Y\,, \ln S)}{Var(\ln S)} = \frac{\beta_1\, \eta_1\, Var(\ln X) + Cov(Q, \ln Y)}{(\eta_1)^2\, Var(\ln X) + Var(Q)}. \qquad [13]$$

Assuming Equation [7], it then follows from Equation [12] that $\tilde{\beta}_1 = \lambda_1 \beta_1$, which leads to the conclusion that left-side measurement error is unproblematic for estimation as long as the measurement age is among the “right points” of the child’ lifecycle, i.e., as long as $\lambda_1 \cong 1$. In contrast, $\tilde{\beta}_1$ will be affected by an upward (downward) bias if $\lambda_1 > 1$ ($\lambda_1 < 1$).

In turn, assuming Equation [11] it follows from Equation [13] that

$$\check{\beta}_1 = \beta_1 \frac{\eta_1}{(\eta_1)^2 + \frac{Var(Q)}{Var(\ln X)}}. \qquad [14]$$

This equation shows that, as long as the parents’ age is among the “right points” of the parents’ lifecycle, i.e., as long as $\eta_1 \cong 1$, estimation with short-run proxies for long-run parental income is affected by a textbook form of attenuation bias, whose size depends on the relative magnitudes of $Var(Q)$ and $Var(\ln X)$ (i.e., on the size of what I later refer as the “variance ratio”). Values of η_1 larger than one will tend to exacerbate attenuation bias, while values smaller than one will tend to reduce it—and, in fact, if η_1 is small enough, the result may even be amplification bias.

The GEiV model plays two different functions. First, as mentioned earlier, it supplies the current methodological justification for the estimation of the conventional IGE by OLS with proxy variables that satisfy some conditions (Nybom and Stuhler 2016). Indeed, the GEiV model suggests that using measures of economic status pertaining to specific ages should eliminate the bulk of lifecycle bias—while the available evidence (which pertains to men) indicates that estimating IGEs with parents’ and children’s information close to age 40 is the best approach

(Haider and Solon 2006; Böhlmark and Lindquist 2006; Mazumder 2001; Nybom and Stuhler 2016). In the case of attenuation bias the GEiV model, and many analyses predating it (e.g. Solon 1992), suggests pushing $Var(Q)$ down by using a multiyear average of parents' income or earnings, rather than a single-year measure, as the proxy measure S.[6] There is strong evidence that the bias can be substantially reduced this way if the average is computed over enough years, although there is disagreement on how many years are necessary to eliminate the bulk of it (see Mazumder 2005; Chetty et al. 2014: 1582 and Online Appendix E; Mitnik et al. 2015:7-15; Mazumder 2016).

The second function of the GEiV model is that it clearly identifies the various potential sources of bias in OLS estimates of the conventional IGE, and therefore provides an analytical framework to help investigate those biases with the very few datasets where long-run income measures can be constructed. The model plays this function directly, as it points to values of λ_1 and η_1 not close enough to one, and $Var(Q)$ not small enough, as sources of bias—which is what, explicitly or implicitly, the mobility literature has traditionally stressed. But the model also plays its second function indirectly, by virtue of being nested within a more general expression (represented here by Equations [12] and [13]) for the probability limit of the OLS estimator of the conventional IGE with short-run variables. Thus, in contrast to the previous literature, Nybom and Stuhler (2016) have emphasized that bias may be the result of the empirical assumptions of the GEiV model not holding up.

[6] In some cases a multiyear average of the logarithm of parental income has been employed instead. This is equivalent to using the geometric mean of parental income over those years as the proxy measure.

For future reference, it is convenient to provide more details on the specific argument advanced by Nybom and Stuhler (2016:245-246). They argued that (a) the parameter λ_1 only captures how differences in annual and lifetime income relate on average across children, (b) there are good reasons to expect idiosyncratic deviations from this average relationship to correlate within families or with parental income, and (c) as a result, we should not expect that $Cov(V, \ln X) = 0$ when $\lambda_1 = 1$, which in turn entails that estimation of the IGE when the latter is the case would not (fully) eliminate left-side lifecycle bias (see Equation [12]). Their empirical analyses with Swedish income data are consistent with this argument, as when $\lambda_1 = 1$ (which happens with their data when the children are about 35 years old) $Cov(V, \ln X) < 0$ and there is what we may call a "residual lifecycle bias" of about – 20 percent.[7] I later refer to Nybom and Stuhler's argument as the "correlated deviations argument."

IV. The GEiVE model

After substituting short-run for long-run income measures in Equation [3], the IGE of the expectation can be estimated using several approaches. These include nonlinear least squares, generalized method of moments (GMM), and pseudo maximum likelihood (PML) based on a variety of distributions in the linear exponential family (e.g., Poisson, gamma, inverse Gaussian).[8] Here I assume that estimation is based on the PPML estimator employed by Mitnik et al. (2015) and Mitnik and Grusky (2017), as there are good reasons to prefer it over other possible estimators of constant-elasticity models (for those reasons, see Santos Silva and

[7] Nybom and Stuhler (2016) also found that lifecycle bias is very close to zero around age 40.

[8] PML estimators are consistent regardless of the actual distribution of the error term, provided that the mean function is correctly specified (Gourieroux, Monfort, and Trognon 1984).

Tenreyro 2006, 2011).

What are the consequences of estimating the IGE of the expectation with the PPML estimator, when short-run proxy variables are substituted for the unavailable long-run variables? To answer, I advance here a measurement-error model aimed at playing for the IGE of the expectation the same role that the GEiV model has played for the estimation of the conventional IGE by OLS. As anticipated in the introduction, developing this model requires addressing difficulties absent in the case of the conventional IGE. First, no closed-form expression for the probability limit of the PPML estimator (or any other estimator that could be used) is available. Second, in nonlinear models the bias induced by right-side measurement error may be in any direction, even when there is only one independent variable and the error is classical and additive in form (e.g., Carroll et al. 2006:Sec. 3.6; Schennach 2016). This is really problematic, as we need to know the direction of the bias, both to develop strategies to reduce it and to be able to interpret results. I address these difficulties by using the relevant population-moment condition and Taylor-series expansions to derive an approximated closed-form expression for the probability limit of the PPML estimator, and by relying on substantive knowledge (e.g., of the signs of various covariances) to conduct proofs and to derive the model's implications. This allows me to determine the direction of all potential biases and the factors driving them.

In the case of left-side measurement error, the assumptions of the GEiVE model I posit are the following:

$$Z_t = \theta_{0t} + \theta_{1t}\, Y + W_t \qquad [15]$$

$$E(W_t) = 0 \qquad [16]$$

$$Cov(W_t, Y) = 0 \qquad [17]$$

$$Cov(W_t, \ln X) = 0, \qquad [18]$$

where $Z_t \geq 0$ is children's income or earnings at age *t*, $Y \geq 0$, $\theta_{0t} + W_t$ is the (additive) measurement error in the short-run measure as a proxy for the long-run measure when $\theta_{1t} = 1$, and θ_{1t} captures left-hand lifecycle bias and thus may be different from one and varies with *t***.**

Focusing now on right-side measurement error, the assumptions are:

$$\ln\ S_k = \pi_{0k} + \pi_{1k} \ln X + P_k \qquad [19]$$

$$E(P_k) = 0 \qquad [20]$$

$$Cov(\ln X\,, P_k) = 0 \qquad [21]$$

$$Cov(P_k, Y) = 0, \qquad [22]$$

where $S_k > 0$ is parents' income or earnings at age *k*, $X > 0$, $\pi_{0k} + P_k$ is the (additive) measurement error in the logarithm of the short-run measure as a proxy for the logarithm of the long-run measure when $\pi_{1k} = 1$, and π_{1t} captures right-hand lifecycle bias and thus may be different from one and varies with t. (I omit from here on the subscripts *t* and *k* to simplify notation.)

It is apparent that the assumptions of the GEiVE model are structurally equivalent to those of the GEiV model. Although I use a different notation for the sake of clarity, Equations [19]-[21] are identical to Equations [8]-[10]. Equations [15]-[17] are different from Equations [5]-[7], as they define a linear projection of Z on Y (instead of a linear projection of $\ln Z$ on $\ln Y$). As a result, unlike in the case of the GEiV model in which $Z_t > 0$ and $Y > 0$, in the GEiVE model $Z_t \geq 0$ and $Y \geq 0$, i.e., as the left-side measurement error pertains to the income variable rather than its logarithm, the model does not preclude that the income variables are zero. Also, the assumptions specified by Equations [18] and [22] are not exactly the same as their GEiV counterparts. Nevertheless, the structural equivalence is clear: While Equations [15]-[17] and

[19]-21] are true by construction, Equations [18] and [22] are empirical assumptions taken to be approximately correct.[9]

The analyses of the GEiV model compare (a) the probability limit of the OLS estimator of the conventional IGE with long-run income variables, with (b) the probability limit of the estimator when short-run proxy variables are used instead. Conducting this comparison is unproblematic, as it relies on the closed-form expression for the probability limit of the slope of an OLS regression. In the case of the PPML estimator the probability limit of the slope is not available in closed form, but it is easy to show that α_1 (the probability limit of the PPML estimator $\hat{\alpha}_1$) solves the population-moment condition

$$\frac{E(X^{\alpha_1} \ln X)}{E(X^{\alpha_1})} = \frac{E(Y \ln X)}{E(Y)} \qquad [23]$$

(Online Appendix, A). This suggests carrying out the analysis in terms of moment conditions. Although this is a feasible approach, it leads to long and convoluted proofs and makes the interpretation of results less straightforward. For this reason, I resort here to approximated closed-form expressions based on moment conditions rather than working with the conditions themselves.

Without any loss of generality, I assume in what follows that $E(Z) = E(Y) = E(\ln S) = E(\ln X) = 1$.[10] An approximated expression for α_1 can be obtained by replacing the expectations

[9] The GEiVE model also makes a third empirical assumption, i.e., $Cov(W_t, P_k) = 0$ for any t and k for which $\theta_{1t} \cong \pi_{1k} \cong 1$. I did not include it above for the same reason I relegated to a footnote the corresponding assumption of the GEiV model (see ftn. 5).

[10] This entails no loss of generality because it can always be achieved by simply changing the monetary units used to measure income, i.e., by dividing each children's income variable by its

on the left-hand side of Equation [23] by second-order Taylor-series approximations, and then solving the quadratic polynomial on α_1 that results. This yields (Online Appendix, A):

$$\alpha_1 \cong C_{\alpha_1} - \left[\left(C_{\alpha_1}\right)^2 - V_{\alpha_1}\right]^{\frac{1}{2}}, \qquad [24]$$

where $C_{\alpha_1} = [Cov(Y, \ln X\,)]^{-1}$, $V_{\alpha_1} = 2\,[Var(\ln X)]^{-1}$, $\left(C_{\alpha_1}\right)^2 > V_{\alpha_1}$, $\frac{\partial \alpha_1}{\partial Cov(Y, \ln X\,)} > 0$, and $\frac{\partial \alpha_1}{\partial\, Var(\ln X)} < 0$.

For simplicity, and as in Haider and Solon (2006), I consider left- and right-side biases separately. I start with the analysis of left-side lifecycle bias. Substituting Z for Y in Equation [3] yields the PRF $\ln E(Z|x) = \tilde{\alpha}_0 + \tilde{\alpha}_1 \ln X$. The probability limit of the PPML estimator, $\tilde{\alpha}_1$, is:

$$\tilde{\alpha}_1 \cong C_{\tilde{\alpha}_1} - \left[\left(C_{\tilde{\alpha}_1}\right)^2 - V_{\alpha_1}\right]^{\frac{1}{2}}, \qquad [25]$$

where:

$$C_{\tilde{\alpha}_1} = \frac{1}{Cov(Z, \ln X\,)} = \frac{1}{\theta_1 Cov(\,Y, \ln X\,) + Cov(W, \ln X\,)},$$

$\left(C_{\tilde{\alpha}_1}\right)^2 > V_{\alpha_1}$, $\frac{\partial \tilde{\alpha}_1}{Cov(Y, \ln X)} > 0$, $\frac{\partial \tilde{\alpha}_1}{\partial\, Cov(W, \ln X)} > 0$, $\frac{\partial \tilde{\alpha}_1}{\partial \theta_1} > 0$, and $\frac{\partial \tilde{\alpha}_1}{\partial\, Var(\ln X)} < 0$.

Equation [25] is the counterpart to Equation [12]. The former equation identifies for the IGE of the expectation, as the latter does for the conventional IGE, the factors that may generate left-side bias—and, in fact, together with Equation [24] it offers a full characterization of the relationship between α_1 and $\tilde{\alpha}_1$, while Equation [12] ignores selection bias and thus fails to do the same for the relationship between β_1 and $\tilde{\beta}_1$. In particular, Equation [25] shows that lifecycle bias is fully eliminated if $\theta_1 = 1$ and $Cov(W, \ln X) = 0$. Indeed, comparing this equation with

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mean, and each parental income variable by the exponential of the mean of its logarithmic values minus one.

Equation [24] indicates that $\tilde{\alpha}_1$ and α_1 are approximated by the same Taylor-series-based expression when those conditions hold. Therefore, under the GEiVE model's empirical assumption specified by Equation [18]—and in strict analogy with what the GEiV model entails for the conventional IGE—it follows from Equation [25] that (the bulk of) left-side lifecycle bias will be eliminated as long as the measurement age is among the "right points" of the children's lifecycle, i.e., as long as $\theta_1 \cong 1$. Moreover, given that income-age profiles vary across people with different levels of human capital and that the latter are strongly associated to parental income, we expect θ_1 to be smaller than one when the children are younger and larger than one when they are older. Therefore, as $\frac{\partial \tilde{\alpha}_1}{\partial \theta_1} > 0$, we also expect that estimation with measures pertaining to when the children are too young (too old) will lead to underestimation (overestimation) of the IGE of the expectation, which is again the same as what the GEiV model entails for the conventional IGE (see the Online Appendix, B, for additional comments and for proofs for claims made in the last two paragraphs).

Moving now to right-side biases, substituting S for X in Equation [3] yields the PRF $\ln E(Y|s) = \breve{\alpha}_0 + \breve{\alpha}_1 \ln S$. The probability limit of the PPML estimator, $\breve{\alpha}_1$, is (Online Appendix, C):

$$\breve{\alpha}_1 \cong C_{\breve{\alpha}_1} - \left[\left(C_{\breve{\alpha}_1}\right)^2 - V_{\breve{\alpha}_1}\right]^{\frac{1}{2}}, \qquad [26]$$

where

$$C_{\breve{\alpha}_1} = [Cov(Y, \ln S\,)]^{-1} = [\pi_1\, Cov(Y, \ln X\,) + Cov(Y, P\,)]^{-1}$$

$$V_{\breve{\alpha}_1} = 2\,[Var(\ln S)]^{-1} = 2\,\{\,Var(\ln X)[(\pi_1)^2 + VR]\}^{-1},$$

$VR = \frac{Var(P)}{Var(\ln X)}$ is the "variance ratio," $\left(C_{\breve{\alpha}_1}\right)^2 > V_{\breve{\alpha}_1}$, $\frac{\partial \breve{\alpha}_1}{Cov(Y, \ln X\,)} > 0$, $\frac{\partial \breve{\alpha}_1}{Cov(Y, P\,)} > 0$, $\frac{\partial \breve{\alpha}_1}{\partial\, Var(\ln X)} < 0$, $\frac{\partial \breve{\alpha}_1}{\partial\, VR} < 0$, and $\frac{\partial \breve{\alpha}_1}{\partial \pi_1} < 0$ (under some conditions specified below). Further, under the GEiVE model's empirical assumption specified by Equation [22], $C_{\breve{\alpha}_1}$ reduces to

$$C_{\breve{\alpha}_1} = [\pi_1\, Cov(Y, \ln X\,)]^{-1}.$$

Equation [26] is the counterpart to Equations [13] and [14]. Similarly to what Equation [13] does for the conventional IGE, Equation [26] identifies the various right-side factors that may jointly affect estimation of the IGE of the expectation with short-run measures of parental income. Likewise, it indicates—as does Equation [14] in the case of the conventional IGE—that even when Equation [22] holds, estimation of the IGE of the expectation with short-run measures of parental income is still compatible with both attenuation and amplification bias, as $(\pi_1)^2 + VR$ may be larger or smaller than one when $\pi_1 < 1$.

There is evidence (e.g., Mazumder 2001; Mazumder 2005) that $Var(P)$ follows an asymmetric U pattern, reaching its minimum value close to age 40 and increasing significantly as parents get older; the variance ratio can be expected to exhibit the same pattern. In addition, for the same reason as in the left-side analysis, we expect π_1 to be smaller than one when the parents are younger and larger than one when they are older. As $\frac{\partial \breve{\alpha}_1}{\partial VR} < 0$ and a sufficient condition for $\frac{\partial \breve{\alpha}_1}{\partial \pi_1} < 0$ is that π_1 is not too small and $VR \leq 1$, estimation with measures pertaining to when the parents are "too old" should lead to underestimating the IGE of the expectation, while using measures pertaining to when they are "too young" may lead to over or underestimating the IGE, depending on the exact values of VR and π_1. All this is, again, exactly as in the case of the GEiV model.

Crucially, estimation with short-run proxies for long-run parental income will be affected by attenuation bias when $\pi_1 \cong 1$. Indeed, when this equality holds exactly $C_{\breve{\alpha}_1} = C_{\alpha_1}$ and $V_{\breve{\alpha}_1}$ becomes:

$$V_{\breve{\alpha}_1} = 2\,\{Var(\ln X)[1 + VR]\}^{-1}.$$

Therefore $\breve{\alpha}_1 < \alpha_1$ and there is attenuation bias, with the magnitude of the bias determined by the size of the variance ratio (see Online Appendix, C, for additional comments and for proofs for claims in the last three paragraphs).

The GEiVE model plays for the IGE of the expectation the same functions that the GEiV model plays for the conventional IGE. First, together with the "nesting equations" (Equations [25] and [26]), it identifies the various potential sources of bias in estimates of the IGE of the expectation with the PPML estimator, and supplies an analytical framework to help investigate those biases. Second, it provides a methodological justification for estimating the IGE of the expectation with proxy variables that satisfy some conditions. With regards to attenuation bias, the GEiVE model suggests that researchers use averages of parental income or earnings over several years, rather than annual measures, so as to reduce $Var(\mathrm{P})$ as much as possible (thus providing an IGE-of-the-expectation counterpart to the approach most often used with the conventional IGE). To minimize lifecycle biases, the GEiVE model suggests using measures of children's and parents' economic status pertaining to ages in which $\theta_1 \cong 1$ and $\pi_1 \cong 1$, respectively.

V. Empirical analyses

Data and estimation

The empirical analyses of this section are based on two PSID samples. The first sample is geared towards studying patterns and magnitudes of left-side lifecycle biases as well as testing the GEiVE model's account of those biases, both of which require obtaining estimates of the IGE of the expectation based on measures of children's long-run income. In line with this requirement, this sample includes information on children born between 1952 and 1958 and present in the PSID in income year 2012.[11] Any year between 1979 and 2012 in which a member of these cohorts was a household head or a household head's spouse, and was between 26 and 55 years old, is included in the sample, that is, the observations are children-years. The sample has observations on 785 children, each of which is observed 21 times on average. The children are not observed every year both because the PSID only reports the needed income information for household heads and spouses (or long-term partners), rather than for all children, and because starting in (survey) year 1997 the PSID has been fielded biannually. I use information on children's total family income (as adults) and on sons' individual earnings in all years they are available, and on parents' average total family income and age when the children were 15-17 years old. I constructed approximate long-run measures by averaging the children's income and earnings information across all available years.

The second sample is geared towards studying patterns and magnitudes of right-side biases. This task requires estimating the IGE of the expectation with measures of parental income obtained by averaging many years of parental information. In accordance with this requirement the sample includes information on children born between 1966 and 1974, for which 25 years of parental data—when the children were between 1 and 25 years old—are available.

[11] The PSID collects income information pertaining to the previous calendar year. From now on, I leave the "income-year" qualifier implicit. Children's ages always refer to their ages in income (rather than survey) years.

The average age of parents when the children are 13 years old is very close to 40; later I refer to this fact by saying that the parental information is “centered around age 40.” As in the first sample, the observations in this sample are children-years. Any year in which a member of the 1966-1974 cohorts was observed in the PSID, and was between 24 and 38 years old, is included in the sample. The sample has observations on 1,274 children, each of which is observed 6.7 times on average. In most analyses, however, I use a subsample pertaining to ages 36-38 (I rely on the larger sample in analyses reported in the Online Appendix). I use information on children’s total family income (as adults) and sons’ individual earnings, and on parents’ average family income and age when the children were 1-25 years old. I use the latter information to construct approximate measures of long-run parental income. Table 1 presents descriptive statistics for both samples. Section D of the Online Appendix provides additional details on the samples and variables.

I estimate the IGE of the expectation with the PPML estimator, and employ sampling weights and compute cluster-corrected robust standard errors in all cases. As the relationship between long-run and short-run measures of income and earnings varies with the age at measurement (both for parents and children), it is customary to include polynomials on children’s and parents’ ages as controls when estimation is based on short-run measures. However, as all IGE estimates I report pertain to subsamples of the samples described above, and in those subsamples the variation in children’s ages is quite small, controlling for children’s age is unnecessary. Mitnik et al. (2015:34) have argued that the age at which parents have their children is not exogenous to their income, that parental age is causally relevant for their children’s life chances, and that insofar as we want persistence measures to reflect the gross association between parental and children’s income we should not control for parental age. Here

I present estimates from models without controls for parental age, but estimates from models with such controls are very similar (and are reported in the Online Appendix).

My main empirical results are presented and discussed in the next two subsections. Additional results, pertaining to interactions between left-side and right-side sources of bias, are available in Section E of the Online Appendix.

Left-side lifecycle bias

Figures 1 and 2 present evidence on left-side lifecycle bias, and its determinants, for the IGEs of expected family income and earnings (in the case of earnings I focus on sons, as is customary). In these figures, the long-run IGE is the IGE of the approximate long-run family income of children (Figure 1) or long-run earnings of sons (Figure 2) with respect to average parental income when the children were 15-17 years old. For the purposes of the analysis here, the latter income is assumed to be the true long-run parental income. The estimated long-run IGE values of 0.4 (family income) and 0.39 (earnings) are undoubtedly affected by right-side attenuation bias (as the parental measure is based on only three years of information). But given that I use the same measure of parental income to compute long-run and short-run IGEs (i.e., IGEs based on the long-run and the short-run measures of children's income or earnings, respectively), it is still possible to draw clear conclusions regarding left-side lifecycle bias.

The figures include four panels. In each of these panels, estimation is based on different sets of subsamples. In the upper-left panel of each figure, the subsamples only include children-income observations pertaining to the exact age indicated in the horizontal axis. In the other three panels estimates are based on progressively larger samples, which include observations pertaining to three-, five-, and seven-year intervals centered on the target age. As more observations are included in the subsamples, short-run point estimates are less affected by sampling variability and the age-profile of the left-side lifecycle bias becomes progressively clearer.

The middle curves in the four panels of Figures 1 and 2 show the relationship between short-run IGEs and children's ages.[12] Estimates vary markedly with the age at which children's incomes and sons' earnings are observed, ranging from 0.23 to 0.45 in the case of family income, and from 0.15 to 0.47 in the case of earnings; these ranges of estimates entail quite different levels of economic persistence. The shape of the curves is consistent with expectations. For both family income and earnings, short-run IGEs based on children's information from their mid-twenties greatly underestimate the long-run IGE, while the downward bias decreases steadily as the information is obtained closer to their late thirties. IGEs based on information pertaining to their late thirties and early forties appear nearly free of lifecycle bias, while those based on information past age 45 are affected by progressively larger upward biases; the latter, however, are comparatively smaller than those that result from using income information pertaining to young ages, as evidenced by the fact that the curves are substantially steeper before age 38 than after age 45 (this is clearest in the bottom-right panels, where the curves are smoothest). Lastly, comparing the four panels within each figure indicates that, at least with small samples like those employed here, short-run estimates based on one year of children's information may be affected by relatively large upward or downward biases even when that information is obtained close to

[12] All estimates underlying these curves, and the corresponding standard errors, can be found in Tables E1 and E2 of the Online Appendix; these tables also include IGE estimates from models with controls for parental age.

age 40, and that these biases can be reduced substantially, if not eliminated, by combining income information from a few years around that age.[13]

The top curve in each panel of Figures 1 and 2 shows how θ_1 changes with the children's age. The parameter increases with age up to their early fifties—monotonically in all but the exact-age curves, which are quite noisy—at which point the curves turn downward. Consistent with expectations, the values of θ_1 are much smaller than one when the children are younger and much larger than one when they are older (up to their early or mid-fifties). The parameter is equal to one around ages 41 (family income) and 39 (earnings). The GEiVE model predicts that at $\theta_1 = 1$ the short-run IGE will be approximately equal to the long-run IGE, and that the former will tend to be smaller (larger) than the latter when $\theta_1 < 1$ ($\theta_1 > 1$). The prediction that the bulk of left-side lifecycle bias will disappear when $\theta_1 = 1$ is derived under the fallible assumption that $Cov(W, X) \cong 0$ at that value of θ_1. Moreover, Nybom and Stuhler's (2016) correlated-deviations argument also applies (mutatis mutandis) in the present context, thus providing a positive reason to doubt the appropriateness of that assumption. The assumption nevertheless holds with a high degree of approximation, as indicated by the bottom curves in all four panels of the figures; indeed, these curves show that when $\theta_1 = 1$, $Corr(\exp(W), X)$ is very close to zero in all cases.[14] As a result, the residual lifecycle biases when $\theta_1 = 1$ are in the -1.6 to 5.4 percent

[13] A similar point was made by Nybom and Stuhler (2016:264) for the conventional IGE. They suggested averaging children's income information across years (within children), which has the same effect as pooling years of information into one sample (as I do here).

[14] I substituted the correlation for the covariance for scaling purposes, and because this made it easier to include all curves in one graph.

range for family income and in the 1.0 to 8.4 percent range for earnings (substantially less, in absolute value, than the residual bias of – 20 percent reported by Nybom and Stuhler for the IGE of the geometric mean). The figures also show that when $\theta_1 < 1$ the short-run IGEs tend to underestimate the long-run IGE, while the opposite happens when $\theta_1 > 1$, as expected. Importantly, short-run IGE estimates at ages in which θ_1 is somewhat larger or smaller than one exhibit a relatively small lifecycle bias. This is important from a practical point of view, as it suggests that IGE estimates based on measures of children's income or earnings obtained close to age 40 will exhibit a small amount of bias even if θ_1 is not exactly one (of course, provided that the samples are large enough).

Right-side biases

Figure 3 presents evidence on right-side lifecycle bias and its determinants, for the IGE of children's family income (left panel) and sons' earnings (right panel). Using a strategy similar to that used in the left-side analysis, in this figure the long-run IGEs are computed with children's income information at ages 36-38 and the approximate long-run parental income. In each panel, the middle curve shows the relationship between short-run IGEs, estimated with parental-income measures based on five years of information, and parental age.[15] The estimates vary significantly across ages, covering the ranges 0.43-0.60 (family income) and 0.48-0.65 (earnings). Both the location and shape of the short-run IGE curves are consistent with expectations. In the case of family income, the IGE curve first rises with age, up to age 34, and

[15] All estimates underlying these curves, and the corresponding standard errors, can be found in Tables A3 and A4 of the Online Appendix. Those tables also include estimates generated with measures of parental income based on 1 to 25 years of information, at various parental ages.

then decreases nearly monotonically up to age 50. In the case of earnings, the IGE estimates do not vary as much up to age 44, but fall very rapidly after that. Each curve is always below the corresponding long-run IGE line, reflecting the attenuation effect associated to using five years of parental information. Nevertheless, in the case of family income the bias is close to nil around age 34. This suggests that amplification bias—mentioned as a theoretical possibility when I presented the GEiVE model—may actually occur at younger ages if the parental income measure is based on more than five years of information. This is indeed the case (see the Online Appendix, Tables E3 and E4), but the observed biases are very small, e.g., about 2 percent, at age 33, for the IGE of family income estimated with nine years of information (compared to that estimated with 25 years at age 40).

As expected, the π_1 curves increase monotonically with parental age while the variance-ratio curves exhibit an asymmetric-U shape. As $Corr(P, Y)$ is close to zero at all parental ages, the amount of lifecycle bias we observe at each age is mostly determined by π_1 and the variance ratio. Although the interaction between these two may lead to different shapes for the short-run IGE curve—as is apparent from comparing the family-income and earnings IGE curves in Figure 3—the fact that π_1 and the variance ratio both increase when the parents are older suggests that, typically, IGE estimates will fall markedly, or at least more rapidly, when parental income pertains to older ages. The evidence in the figure suggests that the variance ratio starts to increase, and the IGE estimates starts to fall more rapidly, when the parents are in their mid-forties.

The GEiVE model entails that when $\pi_1 = 1$ the short-run IGE will be necessarily affected by attenuation bias. This prediction is derived under the fallible assumption that $Cov(P, Y) \cong 0$ at that value of π_1. The prediction is confirmed in Figure 3, where $\pi_1 = 1$ around age 37. However, this fact is not particularly informative, as all short-run estimates in the figure are affected by attenuation bias, regardless of parental age. As indicated above, this is not necessarily the case when measures based on more years of information are employed, which therefore provide a stronger test. Results included in Table 2, which are discussed below, show that IGE estimates are always affected by attenuation bias when $\pi_1 = 1$, regardless of the number of years

The standard approach utilized to assess the extent of attenuation bias that results from the use of short-run parental income, and to ascertain how many years of parental information are needed to eliminate the bulk of that bias, involves (a) estimating the IGE with parental-income measures based on progressively more years of information centered at age 40 (or some other fixed age), and (b) comparing these short-run estimates with the corresponding long-run estimate or, much more commonly, to an asymptotic IGE value extrapolated from the pattern of the short-run estimates.[16] This is an useful approach, but it has one limitation: As the age at which $\pi_1 = 1$ may be different from 40 and may vary with the number of years of information used to compute parental income, the measured attenuation bias most likely reflects not only the effect of the years of information employed but also a "lifecycle effect," i.e., the effect of π_1 being different from one. An alternative approach, which isolates the effect of adding additional

[16] The comparison with an asymptotic value is most often implicit in arguments along the lines that the fact that the differences between IGE estimates based on n, $n + 1$ and $n + 2$ years of information are small and decreasing means that $n + 2$ years of information are enough to eliminate the bulk of attenuation bias.

years of information and which can only be implemented when a long-run measure of parental income is available, is to keep π_1 fixed at one (rather than fixing the parents' age).

Figure 4 and Table 2, which present the results obtained under both approaches, provide clear evidence on the patterns and magnitudes of attenuation bias in the estimation of the IGE of the expectation. The panels on the left of the figure show how the estimates of the family-income IGE (top panel) and earnings IGE (bottom panel) change as more years of information are used, while the panels on the right show the corresponding values of the variance ratio. Table 2 displays the magnitudes of the attenuation biases, expressed as percentages, and the values of π_1 under the first approach and of parental age under the second.

With one year of parental information $Var(P)$ is larger than $Var(\ln X)$ under both approaches, and the variance ratios are very substantial in magnitude. Partly for this reason, the attenuation bias is nearly 49 percent (family income) and 59 percent (earnings) under the standard approach. However, these biases also reflect substantial lifecycle effects, as with one year of information the value of π_1 at age 40 is significantly larger than one. The biases fall to about 25 percent (family income) and 37 percent (earnings) with the alternative approach (as π_1 is equal to one around age 35 rather than 40).

Assessing the magnitude of attenuation biases when parental income measures are based on three and on five years of information is important, as empirical research is quite often based on such measures. There is a very large fall in the variance ratios, and a substantial shrinkage of the values of π_1 towards one (of course, only under the standard approach), when three years of information are used instead of one; moreover, the variance ratio is always substantially smaller than one if at least three years are employed, and decreases steadily as more years of information are added, under both approaches. The large fall in the variance ratio when moving from one to three years of information, together with the concomitant shift of π_1 towards one under the standard approach, translate into a large increase in IGE estimates, and is accompanied by a large reduction in the differences in results across approaches. Nevertheless, with three years of information the attenuation bias is still about 20 percent under the standard approach (for both family income and earnings), and about 13 percent (family income) and 23 percent (earnings) under the alternative approach. With five years of information, attenuation bias is around 15 percent with the former approach (both income measures), and about 8 percent (family income) and 18 percent (earnings) with the latter approach.

As indicated earlier, the issue of how many years of parental information are needed to eliminate the bulk of attenuation bias has been a central concern in the literature (e.g., Mazumder 2005; Mazumder 2016). Stipulating that this happens when attenuation bias is smaller than five percent, the results of Table 2 suggests that, in the case of the IGE of the expectation, achieving this goal with survey data requires employing at least 13 years of parental information (when parental age is measured around age 40).[17]

It is possible, however, that fewer years of information are needed with tax and other administrative data, as Chetty et al. (2014) claimed to be the case for the conventional IGE. One possibility is that administrative data are less affected by measurement error (i.e., that $Var(P)$ is smaller with these data). Another reason, which does not seem to have been discussed previously in the literature, is that tax-based data cover much better than survey data the right tail of income distributions (Fixler and Johnson 2014). Therefore, we can expect $Var(\ln X)$ to be significantly

[17] In the case of earnings this conclusion is based on computing moving averages, each covering three consecutive estimates (e.g., those pertaining to 1, 3 and 5 years of parental information).

larger with the former than with the latter data. An attenuation-bias analysis by Mitnik et al. (2015: Appendix A) nevertheless suggests that at least nine years are necessary with administrative data, as their report concluded that although estimates appeared to be reaching a plateau by year nine, it couldn't be ruled out that they would grow further if even more years were used.

VI. Conclusion

The IGE estimated in the mobility literature has been widely misinterpreted as pertaining to the expectation of children's income, when it actually pertains to its geometric mean. Moreover, the (implicit) reliance on the geometric mean to index conditional income distributions makes studying gender and marriage dynamics in intergenerational processes a very difficult enterprise and leads to IGE estimates affected by substantial selection biases. For these reasons, Mitnik and Grusky (2017) have called for replacing the conventional IGE by the IGE of the expectation. To make this possible, mobility scholars need to have available a measurement-error model that plays for the estimation of the IGE of the expectation the role that Haider and Solon's (2006) GEIV model has played for the estimation of the conventional IGE.

After deriving an approximated closed-form expression for the probability limit of the PPML estimator—something that may be useful beyond the context of the current paper—I have advanced here the needed formal model, that is, a generalized error-in-variables model for the estimation of the IGE of the expectation with short-run income variables. The GEIVE model provides a joint analysis of the biases that may affect estimation of the IGE of the expectation with short-run income measures, and supplies a methodological justification for the estimation of that IGE with the PPML estimator and proxy variables that satisfy some conditions.

The results of the empirical analyses with PSID data offer strong support for the account of lifecycle and attenuation biases provided by the GEiVE model. The empirical results also indicate that the strategy most commonly employed to estimate the conventional IGE by OLS—using a multiyear measure of parental income centered close to age 40 and a measure of children's income obtained around that age—also works well for the estimation of the IGE of the expectation with the PPML estimator. Further, the results suggests that—at least with survey data, and similarly to what has been reported for the conventional IGE (Mazumder 2016: Tables 1 and 2)—estimates of the IGE of the expectation with parental measures based on three to five years of information, that is, the measures often used by mobility scholars, would be affected by substantial attenuation bias. In fact, it appears that at least 13 years of information are needed to eliminate the bulk of that bias.

Our methodological knowledge regarding the estimation of the IGE of the expectation with short-run proxy measures needs to be developed further, and in several different directions. Nevertheless, it seems appropriate to conclude that the measurement-error model and the evidence presented in this paper eliminate the key obstacle for making the IGE of the expectation the workhorse elasticity for the study of intergenerational economic mobility.

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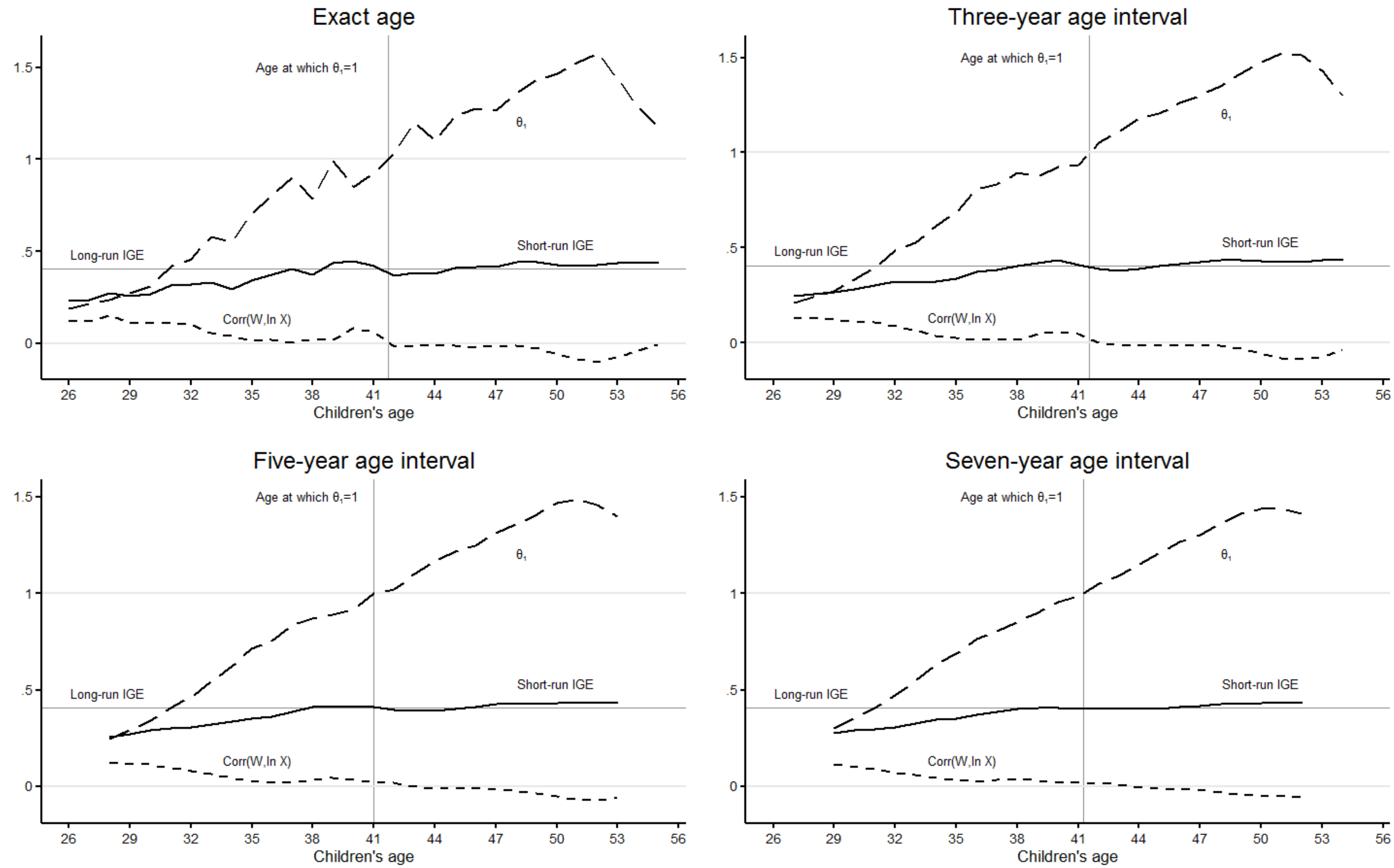
Table 1: Descriptive Statistics (unweighted values)

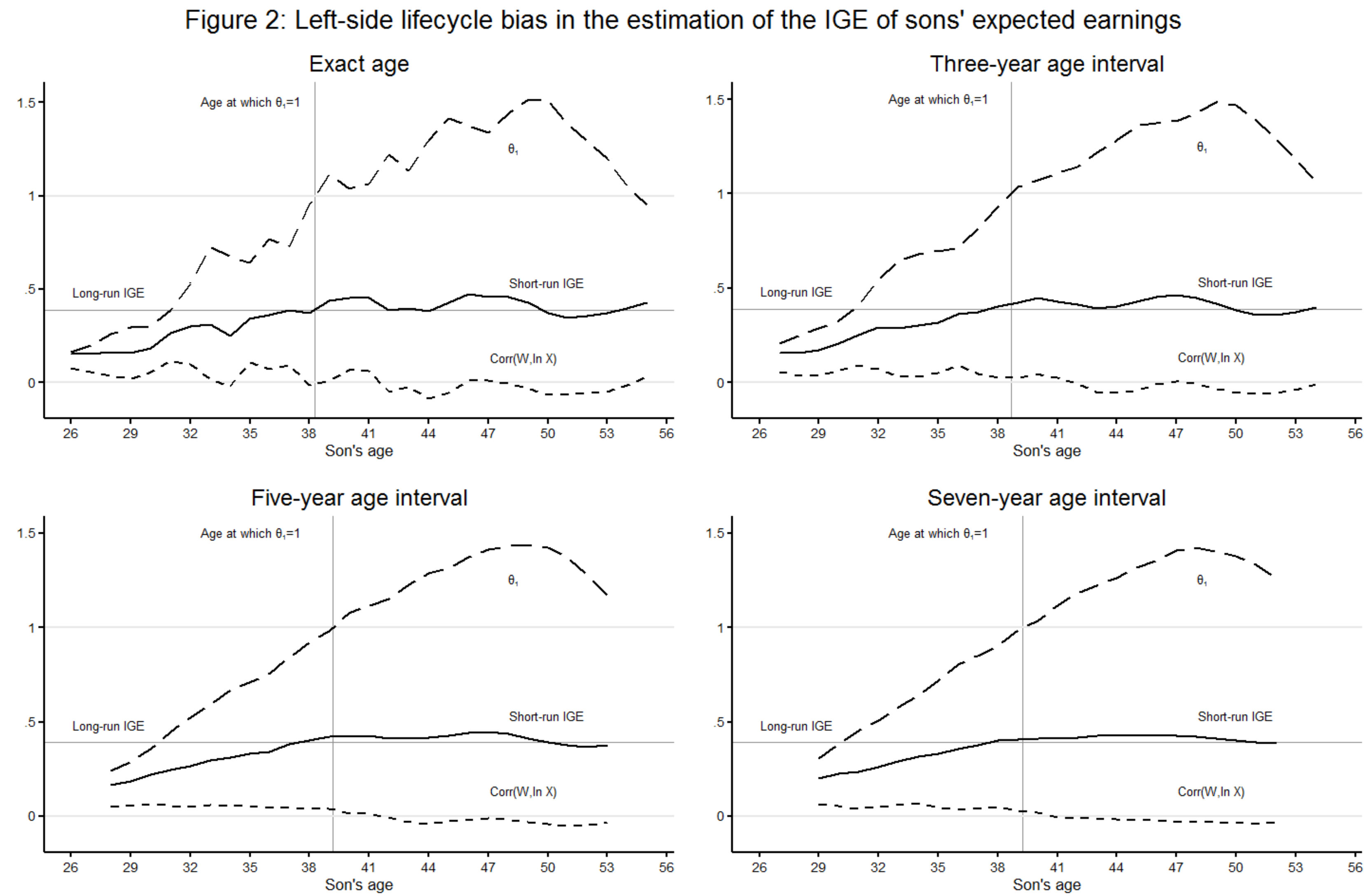
	Sample I: Birth cohorts 1952-1958		Sample II: Birth cohorts 1966-1974	
	All	Sons	All	Sons
Child's gender (% female)	52.2	-	48.6	-
Child's age				
Mean	38.4	38.4	30.5	30.5
Standard deviation	8.3	8.3	4.4	4.4
Child's family income				
Mean	80,845	-	81,586	-
Standard deviation	82,230	-	73,606	-
Child's earnings				
Mean	-	55,213	-	54,768
Standard deviation	-	53,848	-	50,404
Average parental age				
Mean	44.6	44.7	40.4	40.4
Standard deviation	6.8	6.9	6.3	6.2
Average parental income				
Mean	70,395	72,065	88,100	84,377
Standard deviation	54,371	45,501	68,379	56,641
Number of observations	16,497	7,881	8,579	4,121
Number of children	785	377	1,274	655

Monetary values in 2012 dollars (adjusted by inflation using the Consumer Price Index for Urban Consumers - Research Series). In the first sample average parental income and age pertain to when the children were 15-17 years old, while children's family income and sons' earnings pertain to when they were 26-55 years old. In the second sample average parental income and age pertain to when the children were 1-25 years old, while children's family income and sons' earnings pertain to when they were 24-38 years old. The empirical analyses are based on various subsamples of the samples described in this table.

Figure 1: Left-side lifecycle bias in the estimation of the IGE of children's expected family income

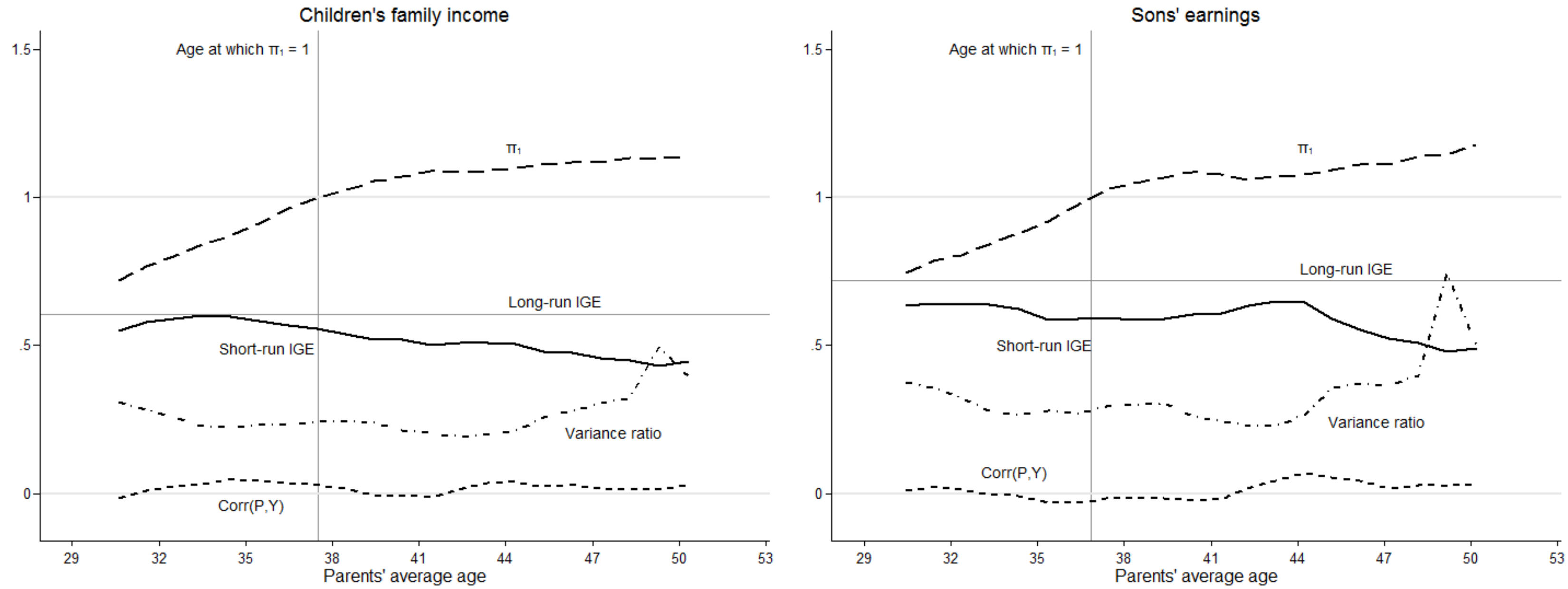


The figure shows the relationship between the short- and long-run IGEs, θ_1, Corr(W, ln X), and children's ages. Parental income is parents' average income when their children were 15-17 years old. The darker-gray horizontal lines indicate the value of the long-run IGE (as approximated by the IGE of the children's average family income at ages 26-55). The vertical lines indicate the ages at which $\theta_1 = 1$ (computed by interpolation). The four panels differ on whether exact ages or three different age intervals are used to select the observations in each estimation sample (the horizontal axis is either the exact age or the mid-point of the age interval).



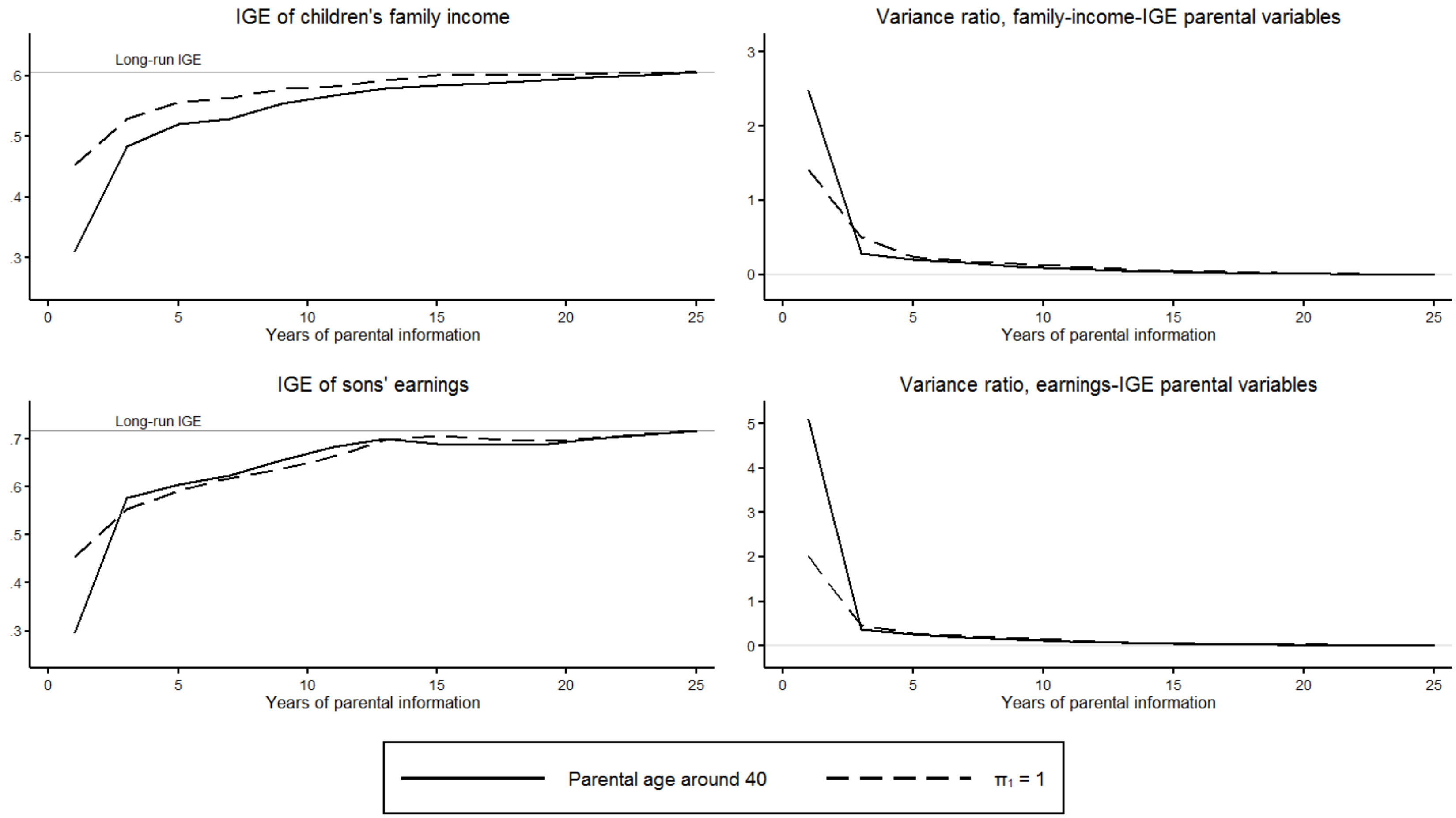
The figure shows the relationship between the short- and long-run IGEs, θ_1, Corr(W, ln X), and children's ages. Parental income is parents' average income when their children were 15-17 years old. The darker-gray horizontal lines indicate the value of the long-run IGE (as approximated by the IGE of the sons' average earnings at ages 26-55). The vertical lines indicate the ages at which $\theta_1 = 1$ (computed by interpolation). The four panels differ on whether exact ages or three different age intervals are used to select the observations in each estimation sample (the horizontal axis is either the exact age or the mid-point of the age interval).

Figure 3: Right-side lifecycle bias in the estimation of the IGE of expected income



The figure shows the relationship between the short- and long-run IGEs, π_1, Corr(P,Y), the variance ratio, and the parents' age at the times their incomes were measured. The panel on the left pertains to the IGE of children's family income when they were 36-38 years old, while the panel on the right pertains to the IGE of sons' earnings at those ages. The horizontal axis is the average age of the parents in the sample. The vertical lines indicate the ages at which $\pi_1 = 1$ (computed by interpolation). The darker-gray horizontal lines indicate values of the long-run IGEs (as approximated by the IGEs computed with parents' information pertaining to when their children where 1-25 years old). Short-run IGEs are based on five years of parental information.

Figure 4: Attenuation bias in the estimation of the IGE of expected income



The panels on the left of the figure show the relationship between IGEs estimated with short-run measures of parental income and the number of years of parental information used to compute those measures, for the IGEs of children's family income (top) and sons' earnings (bottom). The panels on the right of the figure show the relationship between the variance ratio and the years of parental information employed. Each of the four panels includes results from two approaches. The first fixes the age of parents across parental measures; here π_1 may be different from one, and its value may vary across parental measures. The second approach focuses on the IGE estimates that result when π_1 is exactly one; here parental age may vary across parental income measures. These estimates are obtained by interpolation. In the left panels the differences between the long-run IGE and the short-run IGEs estimated under the second approach reflect attenuation bias free of any lifecycle effects, as these are zero by construction. The short-run measures of parental income are based on odd numbers of years of information (1, 3, 5 . . .) and, in the first approach, are centered around the year the children were 13 years old (when mean parental age is about 40). Children's family income refers to when they were 36-38 years old. The(approximated) long-run IGEs are computed with parents' information pertaining to when their children were 1-25 years old.

Online Appendix

Estimating the Intergenerational Elasticity of Expected Income with Short-Run Income Measures: A Generalized Error-in-Variables Model

This appendix includes mathematical proofs and some additional comments about the GEiVE model, additional information on the PSID samples and variables used in the empirical analyses, the estimates underlying the IGE curves presented in Figures 1 to 4, and additional empirical results. The proofs and comments appear in the same order in which the associated results were introduced in the main text. The information about the PSID samples and variables, and the empirical results, are included in the last two sections of the appendix.

I refer to equations presented in the main text using the equation numbers employed there. When equations from the main text are reproduced in this appendix, I rely on their original numbers in the main text.

A. Moment problem solved by the IGE of the expectation and closed-form approximation to the probability limit of the PPML estimator

I derive here the population moment condition specified by Equation [23], and the approximate closed-form expression for α_1 based on second-order Taylor-series approximations (Equation [24]) and related expressions. I also show that the PPML estimator of α_1 solves the sample analog to the moment condition specified by Equation [23].

Equation [3] can be written in additive-error form as:

$$Y = \exp(\alpha_0 + \alpha_1 \ln X) + \Psi, \qquad [A1]$$

where $E(\Psi | \ln x) = 0$. This zero conditional mean assumption entails $E(\Psi \ln X) = 0$. Replacing Ψ by its expression as a function of the parameters results in the following population moment condition:

$$E(Y \ln X) - \exp(\alpha_0) E(X^{\alpha_1} \ln X) = 0.$$

Taking expectation over the population distribution of X in Equation [3], after exponentiating it, leads to $\exp(\alpha_0) = \frac{E(Y)}{E(X^{\alpha_1})}$. Substituting in the last expression gives:

$$\frac{E(X^{\alpha_1} \ln X)}{E(X^{\alpha_1})} = \frac{E(Y \ln X)}{E(Y)}. \qquad [23]$$

The closed-form approximation to α_1 is based on the following second-order Taylor-series approximations around $E(\ln X)$ for the two expectations on the LHS of Equation [23]:

$$E([\exp \ln X]^{\alpha_1} \ln X) \cong [\exp E(\ln X)]^{\alpha_1} E(\ln X) + 0.5\, \alpha_1\, [\exp E(\ln X)]^{\alpha_1} (\alpha_1\, E(\ln X) + 2)\, Var(\ln X)$$

$$E([\exp \ln X]^{\alpha_1}) \cong [\exp E(\ln X)]^{\alpha_1} + 0.5\, [\alpha_1]^2\, [\exp E(\ln X)]^{\alpha_1}\, Var(\ln X).$$

Assuming without any loss of generality that $E(Y) = E(\ln X) = 1$ (see ftn. 10 in the main text), these two approximations become:

$$E([\exp \ln X]^{\alpha_1} \ln X) \cong \exp(\alpha_1) + 0.5\, \alpha_1\, \exp(\alpha_1)\,(\alpha_1 + 2)\, Var(\ln X)$$

$$E([\exp \ln X]^{\alpha_1}) \cong \exp(\alpha_1) + 0.5\, [\alpha_1]^2\, \exp(\alpha_1)\, Var(\ln X).$$

Substituting these expressions on the LHS of Equation [23] and simplifying yields:

$$\frac{1 + 0.5\, \alpha_1\, (\alpha_1 + 2)\, Var(\ln X)}{1 + 0.5\, [\alpha_1]^2\, Var(\ln X)} \cong E(Y \ln X). \qquad [A2]$$

Equation [A2] defines a second-degree polynomial equation on α_1, with solutions:

$$S_1, S_2 = \frac{1}{E(Y \ln X) - 1} \pm \left[\frac{1}{[E(Y \ln X) - 1]^2} - \frac{2}{Var(\ln X)}\right]^{\frac{1}{2}}.$$

As an increase in $Var(\ln X)$, given $E(Y \ln X)$, should be expected to produce a decrease in the value of α_1, the solution with a negative second term is the relevant one. We then have:

$$\alpha_1 \cong \frac{1}{Cov(Y, \ln X) + E(Y)E(\ln X) - 1} - \left[\frac{1}{[Cov(Y, \ln X) + E(Y)E(\ln X) - 1]^2} - \frac{2}{Var(\ln X)}\right]^{\frac{1}{2}}$$

$$\alpha_1 \cong \frac{1}{Cov(Y, \ln X)} - \left[\frac{1}{[Cov(Y, \ln X)]^2} - \frac{2}{Var(\ln X)}\right]^{\frac{1}{2}},$$

which may be written as:

$$\alpha_1 \cong C_{\alpha_1} - \left[\left[C_{\alpha_1}\right]^2 - V_{\alpha_1}\right]^{\frac{1}{2}}, \qquad [24]$$

where

$$C_{\alpha_1} = [Cov(Y, \ln X\ )]^{-1}$$

$$V_{\alpha_1} = 2\, [Var(\ln X)]^{-1}.$$

For Equation [24] to be valid, it cannot be the case that $\left[C_{\alpha_1}\right]^2 < V_{\alpha_1}$. I show next that the opposite inequality holds:

$$\left[C_{\alpha_1}\right]^2 \geq V_{\alpha_1}$$

$$[Cov(Y, \ln X)]^{-2} \geq 2\, [Var(\ln X)]^{-1}$$

$$[Cov(Y, \ln X)]^2 \leq \frac{1}{2}\, Var(\ln X)$$

$$[E(Y \ln X) - 1]^2 \leq \frac{1}{2}\, Var(\ln X).$$

Using now Equation [A2] to substitute an expression for $E(Y \ln X)$ in terms of α_1 in the last inequality:

$$\left[\frac{1 + 0.5\, \alpha_1\, (\alpha_1 + 2)\, Var(\ln X)}{1 + 0.5\, [\alpha_1]^2\, Var(\ln X)} - 1\right]^2 \leq \frac{1}{2}\, Var(\ln X)$$

$$\left[\frac{\alpha_1 Var(\ln X)}{1+\frac{1}{2}\,[\alpha_1]^2\, Var(\ln X)}\right]^2 \leq \frac{1}{2}\, Var(\ln X)$$

$$[\alpha_1]^2[Var(\ln X)]^2 \leq 0.5\, Var(\ln X)\,[1+0.5\,[\alpha_1]^2\, Var(\ln X)]^2$$

$$[\alpha_1]^2 Var(\ln X) \leq \frac{1}{2}+\frac{1}{8}\,[\alpha_1]^4\,[Var(\ln X)]^2+\frac{1}{2}\,[\alpha_1]^2\, Var(\ln X)$$

$$0 \leq \frac{1}{2}+\frac{1}{8}\,[\alpha_1]^4\,[Var(\ln X)]^2-\frac{1}{2}\,[\alpha_1]^2\, Var(\ln X)$$

$$0 \leq 1+\frac{1}{4}\,[\alpha_1]^4\,[Var(\ln X)]^2-\,[\alpha_1]^2\, Var(\ln X).$$

The minimum of the expression on the RHS of the last inequality is zero and occurs when $[\alpha_1]^2\, Var(\ln X) = 2$, so the inequality always holds and $\left[C_{\alpha_1}\right]^2 \geq V_{\alpha_1}$.

In the text I also reported the sign of two derivatives. The derivatives are:

$$\frac{\partial \alpha_1}{\partial Cov(Y, \ln X\,)} = -\frac{1}{[Cov(Y, \ln X)]^2}+\frac{1}{[Cov(Y, \ln X)]^3\,\left[\frac{1}{[Cov(Y,\ln X)]^2}-\frac{2}{Var(\ln X)}\right]^{\frac{1}{2}}}$$

$$= -\frac{1}{[Cov(Y, \ln X)]^2}\left[1-\frac{\frac{1}{Cov(Y,\ln X)}}{\left[\frac{1}{[Cov(Y,\ln X)]^2}-\frac{2}{Var(\ln X)}\right]^{\frac{1}{2}}}\right] > 0$$

$$\frac{\partial \alpha_1}{\partial\, Var(\ln X)} = -\frac{1}{\left[\frac{1}{[Cov(Y,\ln X)]^2}-\frac{2}{Var(\ln X)}\right]^{\frac{1}{2}}[Var(\ln X)]^2} < 0,$$

where I have safely assumed that $Cov(Y, \ln X) > 0$.

So far I have assumed without argument that the probability limit of $\hat{\alpha}_1$, the PPML estimator of the IGE of the expectation, is the population parameter α_1 that solves Equation [23]. A simple justification can be provided by characterizing the PPML estimator as an "analogous estimator." An analogous estimator is a sample statistic that has the same property in the sample as the target parameter has in the population; such an estimator is consistent under quite general conditions (e.g., Manski 1988). The PPML estimator of the IGE of the expectation can be characterized as an analogous estimator of the IGE of the expectation because it solves the sample analog to Equation [23].

Indeed, the first-order conditions solved by the PPML estimator in our context are:

$$\sum\nolimits_1^n [y_i - \exp(\hat{\alpha}_0 + \hat{\alpha}_1 \ln x_i)] = 0$$

$$\sum\nolimits_1^n [y_i - \exp(\hat{\alpha}_0 + \hat{\alpha}_1 \ln x_i)] \ln x_i = 0.$$

Dividing by n, the sample size, and rearranging gives:

$$\bar{y} = \exp(\hat{\alpha}_0)\,\overline{x^{\hat{\alpha}_1}}$$
$$\overline{y_\iota \ln x_\iota} = \exp(\hat{\alpha}_0)\,\overline{x^{\hat{\alpha}_1} \ln x},$$

which can be combined in the obvious way to produce the sample analog to the population condition of Equation [23]:

$$\frac{\overline{x^{\hat{\alpha}_1} \ln x}}{\overline{x^{\hat{\alpha}_1}}} = \frac{\overline{y \ln x}}{\bar{y}}.$$

B. Estimation of the IGE of the expectation with short-run measures of children's economic status and the PPML estimator

I derive here Equation [25], prove related results, and provide some additional comments on the relationship between Equation [25] and Equation [12].

Substituting Z for Y in Equation [3] yields the PRF $\ln E(Z|x) = \tilde{\alpha}_0 + \tilde{\alpha}_1 \ln X$, where $\tilde{\alpha}_1$ is the probability limit of the PPML estimator of the IGE of the expectation with short-run measures of children's income. The approximate closed-form expression for $\tilde{\alpha}_1$ is:

$$\tilde{\alpha}_1 \cong C_{\tilde{\alpha}_1} - \left[\left[C_{\tilde{\alpha}_1}\right]^2 - V_{\alpha_1}\right]^{\frac{1}{2}}, \qquad [25]$$

where $V_{\alpha_1} = 2\,[Var(\ln X)]^{-1}$, $C_{\tilde{\alpha}_1} = \frac{1}{Cov(Z, \ln X)}$, and $\left[C_{\tilde{\alpha}_1}\right]^2 > V_{\alpha_1}$. Using Equation [15] to substitute Z out, $C_{\tilde{\alpha}_1}$ becomes:

$$C_{\tilde{\alpha}_1} = \frac{1}{Cov(\theta_0 + \theta_1 \mathrm{Y} + \mathrm{W}, \ln X)} = \frac{1}{\theta_1\, Cov(\mathrm{Y}, \ln X) + Cov(\mathrm{W}, \ln X)}. \qquad [C2]$$

The last expression for $C_{\tilde{\alpha}_1}$ is the one I reported in the main text as part of Equation [25].

When $\theta_1 = 1$, $C_{\tilde{\alpha}_1}$ reduces to:

$$C_{\tilde{\alpha}_1} = \frac{1}{Cov(Y, \ln X) + Cov(W, \ln X)}.$$

If, in addition, the empirical assumption of the GEiV-E model regarding left-side measurement error (i.e., Equation [18]) also holds, then $C_{\tilde{\alpha}_1}$ further reduces to:

$$C_{\tilde{\alpha}_1} = \frac{1}{Cov(Y, \ln X)} = C_{\alpha},$$

which indicates that there is no left-side lifecycle bias.

In the main text I pointed out that Equation [25] is the IGE-of-the-expectation counterpart of Equation [12]. Like λ_1 and $Cov(V, \ln X)$ in the latter equation (see Nybom and Stuhler 2016), in the former equation values of θ_1 different from one indicate how much income differences among children at the relevant age tend to deviate from the corresponding differences in their long-run incomes, while values of $Cov(W, \ln X)$ different from zero represent both stochastic shocks to children's income and deviations from "average income-age profiles" whose intensity are positively or negatively correlated to the logarithm of their parents' long-run income. As $\frac{\partial \tilde{\alpha}_1}{\partial Cov(W, \ln X)} > 0$ and $\frac{\partial \tilde{\alpha}_1}{\partial \theta_1} > 0$ (see below), it follows that, similarly to analyses carried out with Equation [12] in the case of the conventional IGE:

(a) if $\theta_1 < 1$ ($\theta_1 > 1$) and $Cov(W, \ln X) = 0$, then $\tilde{\alpha}_1 < \alpha_1$ ($\tilde{\alpha}_1 > \alpha_1$), and

(b) if $\theta_1 = 1$ and $Cov(W, \ln X) < 0$ ($Cov(W, \ln X) > 0$), then $\tilde{\alpha}_1 < \alpha_1$ ($\tilde{\alpha}_1 > \alpha_1$).

In addition, as we can expect θ_1 to be smaller than one when the children are younger and larger than one when they are older, estimation with measures pertaining to when the children are too young (too old) should lead to underestimation (overestimation) of the IGE of the expectation, exactly as in the case of the conventional IGE.

I also pointed out that Equation [25] offers a better characterization of left-side potential biases when the IGE of the expectation is estimated with short-run measures that Equation [12] does for the conventional IGE. Indeed, although Equation [12] identifies the biases generated by the violation of the left-side empirical conditions $\lambda_1 = 1$ and $Cov(V, \ln X) = 0$, it neglects the selection bias that almost certainly results from dropping children with zero short-run income (Mitnik and Grusky 2017). Therefore, it offers a characterization of the relationship between $\tilde{\beta}_1$ and β_1 that is not fully correct. As this bias is not an issue for the IGE of the expectation, Equation [25] does attain for that IGE (of course, at the level of approximation allowed by second-order Taylor-series approximations) the goal that Equation [12] does not fully attain for the conventional IGE.

In the main text I reported the sign of four derivatives, some of which I used above. The derivative of $\tilde{\alpha}_1$ with respect to $Var(\ln X)$ is:

$$\frac{\partial \tilde{\alpha}_1}{\partial\, Var(\ln X)} = -\frac{1}{\left[\left(C_{\tilde{\alpha}_1}\right)^2 - \frac{2}{Var(\ln X)}\right]^{\frac{1}{2}} [Var(\ln X)]^2} < 0.$$

It is more convenient to express the other derivatives in terms of the following derivative:

$$\frac{\partial \tilde{\alpha}_1}{\partial C_{\tilde{\alpha}_1}} = \left[1 - \frac{C_{\tilde{\alpha}_1}}{\left[\left(C_{\tilde{\alpha}_1}\right)^2 - \frac{2}{Var(\ln X)}\right]^{1/2}}\right] < 0,$$

for which the sign can be easily established given that $C_{\tilde{\alpha}_1} = \frac{1}{Cov(Z, \ln X)}$ can be safely assumed to be positive. Now I can write the remaining derivatives as:

$$\frac{\partial \tilde{\alpha}_1}{Cov(Y, \ln X)} = \frac{\partial \tilde{\alpha}_1}{\partial C_{\tilde{\alpha}_1}} \frac{\partial C_{\tilde{\alpha}_1}}{\partial\, Cov(Y, \ln X)} = \frac{\partial \tilde{\alpha}_1}{\partial C_{\tilde{\alpha}_1}} \left\{-\frac{\theta_1}{[\theta_1\, Cov(\mathrm{Y}, \ln X) + Cov(\mathrm{W}, \ln X)]^2}\right\} > 0$$

$$\frac{\partial \tilde{\alpha}_1}{\partial\, Cov(W,\ \ln X)} = \frac{\partial \tilde{\alpha}_1}{\partial C_{\tilde{\alpha}_1}} \frac{\partial C_{\tilde{\alpha}_1}}{\partial\, Cov(W,\ \ln X)} = \frac{\partial \tilde{\alpha}_1}{\partial C_{\tilde{\alpha}_1}} \left\{-\frac{1}{[\theta_1\, Cov(\mathrm{Y}, \ln X) + Cov(\mathrm{W}, \ln X)]^2}\right\} > 0$$

$$\frac{\partial \tilde{\alpha}_1}{\partial \theta_1} = \frac{\partial \tilde{\alpha}_1}{\partial C_{\tilde{\alpha}_1}} \frac{\partial C_{\tilde{\alpha}_1}}{\partial\, \theta_1} = \frac{\partial \tilde{\alpha}_1}{\partial C_{\tilde{\alpha}_1}} \left\{-\frac{Cov(\mathrm{Y}, \ln X)}{[\theta_1\, Cov(\mathrm{Y}, \ln X) + Cov(\mathrm{W}, \ln X)]^2}\right\} > 0,$$

where in determining the sign of the first and third derivatives I safely assumed that $\theta_1 > 0$ and $Cov(\mathrm{Y}, \ln X) > 0$, respectively.

C. Estimation of the IGE of the expectation with short-run measures of parents' economic status and the PPML estimator

I derive here Equation [26] and related expressions, and provide additional comments. Substituting S for X in Equation [3] yields the PRF $\ln E(Y|s) = \breve{\alpha}_0 + \breve{\alpha}_1 \ln S$, where $\breve{\alpha}_1$ is the probability limit of the PPML estimator of the IGE of the expectation with short-run measures of parental income. The approximate closed-form expression for $\breve{\alpha}_1$ is:

$$\breve{\alpha}_1 \cong C_{\breve{\alpha}_1} - \left[\left[C_{\breve{\alpha}_1}\right]^2 - V_{\breve{\alpha}_1}\right]^{\frac{1}{2}}, \qquad [26]$$

where $C_{\breve{\alpha}_1} = [Cov(Y, \ln S\,)]^{-1}$, $V_{\breve{\alpha}_1} = 2\,[Var(\ln S)]^{-1}$ and $\left[C_{\breve{\alpha}_1}\right]^2 > V_{\breve{\alpha}_1}$. Using Equation [19] to substitute S out, $C_{\breve{\alpha}_1}$ and $V_{\breve{\alpha}_1}$ become:

$$\begin{aligned} C_{\breve{\alpha}_1} &= [Cov(Y, \pi_0 + \pi_1 \ln X + P)]^{-1} \\ &= [\pi_1\, Cov(Y, \ln X\,) + Cov(Y, P)]^{-1} \\ V_{\breve{\alpha}_1} &= 2\,[Var(\pi_0 + \pi_1 \ln X + P)]^{-1} \\ &= 2\,\{[\pi_1]^2\, Var(\ln X) + Var(P) + 2\, Cov(\ln X, P)\}^{-1} \\ &= 2\,\{[\pi_1]^2\, Var(\ln X) + Var(P)\}^{-1} \\ &= 2\,\{\, Var(\ln X)[(\pi_1)^2 + VR]\}^{-1}, \end{aligned}$$

where $VR = \frac{Var(P)}{Var(\ln X)}$. These are the expressions for $C_{\breve{\alpha}_1}$ and $V_{\breve{\alpha}_1}$ reported in the main text as part of Equation [26].

Equation [26] is the IGE-of-the-expectation counterpart to Equations [13] and [14]. Similarly to Equation [13] in the case of the conventional IGE, Equation [26] identifies the various right-side factors that may jointly affect estimation of the IGE of the expectation when short-run measures of parental income are substituted for their long-run counterparts. These factors are (a) $\pi_1 \neq 1$, (b) $Cov(Y, \exp(P)) \neq 0$, and (c) $VR > 0$ (which is the case by assumption). Values of π_1 different from one reflect how much the differences in short-run parental income across children tend to deviate from the corresponding differences in long-run parental income (when parental income is measured at the relevant parental age).

As pointed out in the main text, under the empirical assumption of the GEiV-E model regarding right-side measurement error specified by Equation [22], $C_{\breve{\alpha}_1}$ reduces to

$$C_{\breve{\alpha}_1} = [\pi_1\, Cov(Y, \ln X\,)]^{-1}.$$

So Equation [26] indicates, as does Equation [14] in the case of the conventional IGE, that even when Equation [22] holds, estimation of the IGE of the expectation with short-run measures of parental income is still compatible with both attenuation and amplification bias—as $VR > 0$ tends to produce attenuation bias, but whether this is the case or not also depends on the value of π_1. As a sufficient condition for $\frac{\partial \breve{\alpha}_1}{\partial \pi_1} < 0$ is that π_1 is not too small and VR is not larger than one (see below), and we can expect π_1 to be smaller than one when the parents are younger and larger than one when they are older, estimation with measures pertaining to when the parents are too young (too old) should contribute to overestimating (underestimating) the IGE of the expectation.

When $\pi_1 = 1$ the differences are, on average across children, similar for the long-run and the short-run measures. In this context the only potential source of bias that remains is $VR > 0$, as $C_{\breve{\alpha}_1} = C_{\alpha_1}$ and $V_{\breve{\alpha}_1}$ becomes:

$$V_{\breve{\alpha}_1} = 2\,\{\,Var(\ln X)[1 + VR]\}^{-1}.$$

Therefore $\breve{\alpha}_1 < \alpha_1$ and there is attenuation bias, with the magnitude of the bias determined by VR, i.e., by the relative sizes of $Var(P)$ and $Var(\ln X)$. Given $Var(\ln X)$, this bias and $Var(P)$ fall together (as $\frac{\partial \breve{\alpha}_1}{\partial\, Var(P)} < 0$, see next), which provides a formal justification for the strategy of using a multiyear average of parental income instead of an annual measure to estimate the IGE of the expectation (an approach already employed, without a formal justification, by Mitnik et al., 2015).

In the text I reported the sign of five derivatives, some of which I used above. The derivatives of $\breve{\alpha}_1$ with respect to $Var(\ln X)$ and $Var(P)$ are:

$$\frac{\partial \breve{\alpha}_1}{\partial\, Var(\ln X)} = - \frac{[\pi_1]^2}{[(\pi_1)^2\, Var(\ln X) + Var(P)]^2 \left\{[\pi_1\, Cov(Y, \ln X\,) + Cov(Y,P)]^2\ - \frac{2}{[\pi_1]^2\, Var(\ln X) + Var(P)}\right\}^{\frac{1}{2}}} < 0$$

$$\frac{\partial \breve{\alpha}_1}{\partial\, Var(P)} = - \frac{1}{[(\pi_1)^2\, Var(\ln X) + Var(P)]^2 \left\{[\pi_1\, Cov(Y, \ln X\,) + Cov(Y,P)]^2\ - \frac{2}{[\pi_1]^2\, Var(\ln X) + Var(P)}\right\}^{\frac{1}{2}}} < 0.$$

From the last derivative, it immediately follows that $\frac{\partial \breve{\alpha}_1}{\partial\, VR} < 0$ as well.

It is more convenient to express two of the other derivatives in terms of the following derivative:

$$\frac{\partial \breve{\alpha}_1}{\partial C_{\breve{\alpha}_1}} = \left[1 - \frac{C_{\breve{\alpha}_1}}{\left[\left(C_{\breve{\alpha}_1}\right)^2 - \frac{2}{[\pi_1]^2\, Var(\ln X) + Var(P)}\right]^{1/2}}\right] < 0,$$

for which the sign can be easily established given that $C_{\breve{\alpha}_1} = \frac{1}{Cov(Y, \ln S\,)}$ can be safely assumed to be positive. The two derivatives of interest can be written as:

$$\frac{\partial \breve{\alpha}_1}{\partial Cov(Y, \ln X\,)} = \frac{\partial \breve{\alpha}_1}{\partial C_{\breve{\alpha}_1}} \frac{\partial C_{\breve{\alpha}_1}}{Cov(Y, \ln X\,)}$$

$$= \frac{\partial \breve{\alpha}_1}{\partial C_{\breve{\alpha}_1}} \left[- \frac{\pi_1}{[\pi_1\, Cov(Y, \ln X\,) + Cov(Y, P)]^2}\right] > 0.$$

$$\frac{\partial \breve{\alpha}_1}{\partial Cov(Y, P\,)} = \frac{\partial \breve{\alpha}_1}{\partial C_{\breve{\alpha}_1}} \frac{\partial C_{\breve{\alpha}_1}}{Cov(Y, P\,)}$$

$$= \frac{\partial \breve{\alpha}_1}{\partial C_{\breve{\alpha}_1}} \left[- \frac{1}{[\pi_1\, Cov(Y, \ln X\,) + Cov(Y, P)]^2}\right] > 0.$$

Under the GEiVE model's empirical assumption specified by Equation [21], i.e., $Cov(Y, P) = 0$, the derivative of $\breve{\alpha}_1$ with respect to π_1 is:

$$\frac{\partial \tilde{\alpha}_1}{\partial \pi_1} = - \frac{1}{\mathrm{Cov}(\mathrm{Y}, \ln X)\, [\pi_1]^2} + \frac{\frac{2}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]^2\, [\pi_1]^3} - \frac{4\, \pi_1 \mathrm{Var}(\ln X)}{[\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)]^2}}{2\left\{\frac{1}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]^2\, [\pi_1]^2} - \frac{2}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)}\right\}^{1/2}}.$$

This derivative is negative if and only if:

$$\frac{\frac{2}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]^2\, [\pi_1]^3} - \frac{4\, \pi_1 \mathrm{Var}(\ln X)}{[\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)]^2}}{2\left\{\frac{1}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]^2\, [\pi_1]^2} - \frac{2}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)}\right\}^{1/2}} < \frac{1}{\mathrm{Cov}(\mathrm{Y}, \ln X)\, [\pi_1]^2}$$

$$\frac{\frac{\mathrm{Cov}(\mathrm{Y}, \ln X)\, [\pi_1]^2}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]^2\, [\pi_1]^3} - \frac{2\, [\pi_1]^2 \mathrm{Var}(\ln X)\, \pi_1 \mathrm{Cov}(\mathrm{Y}, \ln X)}{[\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)]^2}}{\left\{\frac{1}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]^2\, [\pi_1]^2} - \frac{2}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)}\right\}^{1/2}} < 1$$

$$\frac{1}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]\, [\pi_1]} - \frac{2}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)} \frac{\pi_1 \mathrm{Cov}(\mathrm{Y}, \ln X)\, [\pi_1]^2 \mathrm{Var}(\ln X)}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)} < \left\{\frac{1}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]^2\, [\pi_1]^2} - \frac{2}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)}\right\}^{1/2}$$

$$\left\{\frac{1}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]\, [\pi_1]} - \frac{2}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)} \frac{\pi_1\, \mathrm{Cov}(\mathrm{Y}, \ln X)\, [\pi_1]^2\, \mathrm{Var}(\ln X)}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)}\right\}^2 < \frac{1}{[\mathrm{Cov}(\mathrm{Y}, \ln X)]^2\, [\pi_1]^2} - \frac{2}{\mathrm{Var(P)} + [\pi_1]^2 \mathrm{Var}(\ln X)}$$

$$\left\{\frac{2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\frac{\pi_1\,\mathrm{Cov}(\mathrm{Y},\ln X)\,[\pi_1]^2\mathrm{Var}(\ln X)}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\right\}^2 - 2\,\frac{2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\frac{[\pi_1]^2\,\mathrm{Var}(\ln X)}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)} < -\frac{2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}$$

$$-\frac{2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\frac{2\,[\pi_1]^2\,\mathrm{Var}(\ln X)}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)} < -\frac{2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\left\{1-\frac{2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\left[\frac{\pi_1\mathrm{Cov}(\mathrm{Y},\ln X)\,[\pi_1]^2\,\mathrm{Var}(\ln X)}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\right]^2\right\}$$

$$\frac{2\,\mathrm{Var}(\ln X)\;\;[\pi_1]^2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)} > 1-\frac{2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\left[\pi_1\,\mathrm{Cov}(\mathrm{Y},\ln X)\frac{[\pi_1]^2\,\mathrm{Var}(\ln X)}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\right]^2$$

$$\frac{2\,\mathrm{Var}(\ln X)\;\;[\pi_1]^2+2\,\left[\pi_1\,\mathrm{Cov}(\mathrm{Y},\ln X)\frac{[\pi_1]^2\,\mathrm{Var}(\ln X)}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\right]^2}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)} > 1$$

$$2\,[\pi_1]^2\,\mathrm{Var}(\ln X)+2\,\left[\pi_1\,\mathrm{Cov}(\mathrm{Y},\ln X)\frac{[\pi_1]^2\,\mathrm{Var}(\ln X)}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\right]^2 > \mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)$$

$$[\pi_1]^2\,\mathrm{Var}(\ln X)\;+2\,[\pi_1]^2\,[\mathrm{Cov}(\mathrm{Y},\ln X)]^2\,\left[\frac{[\pi_1]^2\,\mathrm{Var}(\ln X)}{\mathrm{Var(P)}+[\pi_1]^2\mathrm{Var}(\ln X)}\right]^2-\mathrm{Var(P)}>0$$

$$[\pi_1]^2\left\{\mathrm{Var}(\ln X)+2\,\left[\frac{\mathrm{Cov}(\mathrm{Y},\ln X)}{1+\frac{1}{[\pi_1]^2}\frac{\mathrm{Var(P)}}{\mathrm{Var}(\ln X)}}\right]^2\right\}-\mathrm{Var(P)}>0.$$

Therefore, safely assuming that $\mathrm{Cov}(\mathrm{Y},\ln X)>0$, a sufficient condition for $\frac{\partial\tilde{\alpha}_1}{\partial\pi_1}<0$ is that π_1 is not too small and $\mathrm{Var(P)}$ is not larger than $\mathrm{Var}(\ln X)$ (i.e., VR is not larger than one). In the empirical analyses, $\mathrm{Var(P)} > Var(\ln X)$ only when the parental measure is based on one year of income information.

This is exactly as in the GEiV model. Under the empirical assumption that $Cov(Q,\ln Y)=0$, from Equation [14] we have:

$$\frac{\partial\tilde{\beta}_1}{\partial\eta_1} = -\frac{2\,\beta_1[\eta_1\mathrm{Var}(\ln X)]^2}{\{\mathrm{Var(Q)}+\;[\eta_1]^2\,\mathrm{Var}(\ln X)\}^2}+\frac{\beta_1\,\mathrm{Var}(\ln X)}{\mathrm{Var(Q)}+[\eta_1]^2\,\mathrm{Var}(\ln X)}$$
$$=\beta_1\frac{\mathrm{Var}(\ln X)\{Var(Q)-[\eta_1]^2\,\mathrm{Var}(\ln X)\}}{\{\mathrm{Var(Q)}+\;[\eta_1]^2\,\mathrm{Var}(\ln X)\}^2}.$$

The last equation shows that a sufficient condition for $\frac{\partial\tilde{\beta}_1}{\partial\eta_1}<0$ is that $\eta_1$is not too small and $\mathrm{Var(Q)}$ is not larger than $\mathrm{Var}(\ln X)$.

D. Additional information on samples and variables

I provide here additional information on the PSID samples and variables employed in the empirical analyses.

Like Hertz (2007), I define "child" broadly to include anyone of the right age reported in the PSID to be either the son, daughter, stepson, stepdaughter, nephew, niece, grandson or granddaughter of the household head or his wife (or long-term partner).[1] As Hertz (2007:35) put it, "the idea is to look at the relation between children's income and the income of the households in which they were raised, even if that household was not, or not always, headed by their mother or father." Similarly, when the children are 1-17 years old, the "father" is the household head (if the head is male), while the "mother" is either the household head (if the head is female) or the head's wife or long-term partner. When the children are older than 17, the father and mother are those who were determined to be the father and mother at age 17 (this is only relevant for the sample used in the right-side analyses).

The PSID switch to collecting data biannually (from survey year1997 on) generated some problems for the analyses conducted in this paper. Because information on children's income and earnings in the years 1997, 1999, 2001, 2003, 2005, 2007, 2009 and 2011 is not available, if the data are used as provided the estimates of short-run IGEs (and other parameters and quantities discussed throughout the paper) at different children's ages would be based on information from different birth cohorts. For instance, in the left-side analyses, the estimate of the IGE when the children are 34 years old would be based on information from all seven cohorts included in the sample (i.e., cohorts 1952 to 1958). The estimate of the IGE at age 40 would be based on six cohorts, as measures of income at age 40 are not available for the 1957 cohort. And the IGE estimates at age 45 and 46 would be based on three and four cohorts, respectively, as measures of income at age 45 are not available for the 1952, 1954, 1956 and 1958 cohorts, while measures of income at age 46 are not available for the 1953, 1955 and 1957 cohorts. So using the data as provided would add a substantial amount of noise to the curves shown in the paper's figures.

To address this problem, I used the children-years from the two years contiguous to each missing year to "represent" the latter, with the sampling weights of those two children-years divided by two and the children's ages adjusted appropriately. As the last year of information available at the time of completing my empirical analyses was 2012 while the children born in 1958 became 55 years old in 2013, for that cohort I represented the year 2013 with the information from 2012, with the age adjusted from 54 to 55. In the case of the right-side analyses, the lack of income information in the years listed above also affected the computation of parental measures for the cohorts 1972, 1973 and 1974 (the other six cohorts

[1] By convention, the PSID never codes a woman as household head and her spouse as "husband."

are not affected). In this case I represented the years 1997 (all three cohorts), 1999 (last two cohorts) and 2001 (1974 cohort) with the average income of the parents in the two contiguous years. The unweighted descriptive statistics presented in Table 1 pertain to the PSID data without the addition of information from contiguous years to represent the children-years that are missing.

Children's annual earnings are measured, in all years, in the way the PSID measured "income from labor" up to 1992. Starting in 1993 the PSID did not include any longer, in its measure of income from labor, the labor portion of business income and 50 percent of farm income, which were included in previous years. So I used the PSID variable up to 1992, but for later years I computed the pre-1993 notion using several variables. The annual measures of family income are based on the PSID notion of "total family income." But as the income components the PSID used to compute total family income are effectively affected by top coding in the period 1970-1978 (i.e., top codes were not only in place but were "binding" in that period for some people), and the PSID-computed total-family income for those years is based on these top-coded values, I proceeded as follows: (a) I addressed the top-coding of all income components in 1970-1978 by using Pareto imputation (Fichtenbaum and Shahidi 1988), and (b) I recomputed total family income for those years with the Pareto-imputed component variables.

In the earnings analyses I estimated elasticities with respect to parental income in spite of the fact that it is more common to estimate them with respect to fathers' earnings. However, as Corak (2006:54) and Mazumder (2005:250) have also pointed out, there are good reasons why it is preferable to use a measure of family income: (a) it incorporates the income of mothers and thus better indexes the full complement of resources available to invest in children; (b) it reflects the ability of families to draw on other income sources in response to transitory earnings shocks; and (c) it avoids any selection bias that may result from omitting sons with absent fathers (as they are likely to be comparatively disadvantaged). For research that estimated the elasticity of sons' earnings with respect to family income, see for instance Berman and Taubman (1990), Chadwick and Solon (2002), Levine and Mazumder (2002), Mazumder (2005), Mitnik et al. (2015), and Mitnik and Grusky (2017).

E. IGE estimates underlying figures, and additional empirical results

Tables E1 and E2 present estimates of the IGE of children's family income and sons' earnings, respectively, at ages 26 to 55. The tables report estimates both from models that include and do not include a quadratic polynomial on parental age on the RHS. The estimates from the latter models are those underlying the short-run IGE curves shown in Figures 1 and 2. As indicated in the main text, IGE estimates from models that do control for parental age are very similar to those that do not control for parental age.

Tables E3 and E4 present estimates of the IGE of children's family income and sons' earnings, respectively, from models using parental-income measures based on 1 to 25 years of income information and pertaining to when the average age of parents ranged from 29 to 52 years old (i.e., when the children were from 1 to 25 years old). The estimates based on five years of information are those underlying the short-run IGE curves shown in Figures 3 and 4. As indicated in the main text, the tables show that amplification (rather than attenuation) bias is not just a theoretical possibility but actually occurs when the parents are young enough and the parental-income measures are based on enough years of information. When amplification bias occurs, it is quite small in all cases (at least for the combinations of parental age and years of parental information that are available with the sample used here).

The analyses in the main text made it clear that it's important to consider the effect of parental years of information on IGE estimates jointly with those of parental age, and the same is likely to be the case for children's age. Figure E1 offers an overview of these joint effects. The top panels show the joint effects of children's age and years of information, for children 24 to 38 years old and 1 to 25 years of parental information. As in Figure 4, switching from one to three years of information brings about a much larger decrease in bias than subsequent additions of years of information do, at all children's ages. However, if we ignore the one-year estimates, the effects of age and years of information on the IGE of family income (left panel) are approximately linear and additive: The R-squared of such a model is 0.98. Alternatively, keeping all estimates but adding to the model's RHS an indicator variable for years of information equal to one produces an R-squared of 0.97. The joint effects on the IGE of earnings (right panel) are still very roughly additive, but they appear more clearly nonlinear (especially in the case of children's age, although this likely results in part from the much smaller samples used to produce the earnings estimates); nevertheless, fitting an additive linear model delivers an R-squared of 0.89 when estimates based on one year of information are excluded, and of 0.92 if they are kept and an indicator variable is added to the model. Tables E5 and E6 present the IGE estimates underlying the top panels (including standard errors), and similar estimates from model that control for parental age. The latter estimates are very similar to the former.

The bottom panels of Figure E1, which are based on information from Tables E3 and E4, show the joint effects of average parental age and years of parental information on IGE estimates, for parental ages 31, 35, 40, 45 and 49, and 1, 3, 5, 7 and 9 years of information. As in previous analyses, switching from one to three years of parental information has a much larger effect on estimates than any subsequent addition of information. The relationship between years of parental information, parental age and IGE estimates is more complex that in the first two panels. The way in which π_1 and VR change with parental age suggests that the relationship between IGE estimates and parental age should be nonlinear and switch sign at some point (from positive to negative), which is consistent with what we see in the figure (more clearly for the IGE of family income than for the IGE of earnings). Models like those fitted for the top panels, but in which parental age enters as a quadratic polynomial, deliver an R-squared of 0.95 (0.94

with an indicator variable for one year of information) in the case of the IGE of family income, and an R-squared of 0.78 (0.90 with an indicator variable) in the case of the IGE of earnings.

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Table E1: IGE of children's expected family income, by children's age

Children's age	Without controlling for parental age				Controlling for parental age			
	Exact age	Three-year interval	Five-year interval	Seven-year interval	Exact age	Three-year interval	Five-year interval	Seven-year interval
26	**0.23**	-	-	-	**0.22**	-	-	-
	(0.05)				(0.05)			
27	**0.24**	**0.25**	-	-	**0.24**	**0.25**	-	-
	(0.05)	(0.04)			(0.05)	(0.05)		
28	**0.28**	**0.26**	**0.25**	-	**0.28**	**0.26**	**0.25**	-
	(0.05)	(0.04)	(0.04)		(0.05)	(0.05)	(0.05)	
29	**0.26**	**0.27**	**0.27**	**0.28**	**0.25**	**0.26**	**0.27**	**0.27**
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
30	**0.27**	**0.28**	**0.29**	**0.29**	**0.26**	**0.28**	**0.28**	**0.28**
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
31	**0.31**	**0.30**	**0.30**	**0.30**	**0.31**	**0.30**	**0.29**	**0.29**
	(0.06)	(0.06)	(0.05)	(0.05)	(0.07)	(0.06)	(0.06)	(0.05)
32	**0.32**	**0.32**	**0.31**	**0.31**	**0.31**	**0.31**	**0.30**	**0.30**
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
33	**0.33**	**0.32**	**0.32**	**0.32**	**0.32**	**0.31**	**0.31**	**0.32**
	(0.07)	(0.06)	(0.06)	(0.06)	(0.07)	(0.06)	(0.06)	(0.06)
34	**0.30**	**0.32**	**0.34**	**0.35**	**0.30**	**0.32**	**0.33**	**0.34**
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
35	**0.34**	**0.34**	**0.35**	**0.35**	**0.34**	**0.33**	**0.34**	**0.34**
	(0.07)	(0.06)	(0.07)	(0.06)	(0.07)	(0.07)	(0.07)	(0.06)
36	**0.37**	**0.38**	**0.36**	**0.37**	**0.37**	**0.37**	**0.35**	**0.36**
	(0.07)	(0.07)	(0.06)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)
37	**0.41**	**0.39**	**0.39**	**0.39**	**0.39**	**0.37**	**0.38**	**0.38**
	(0.08)	(0.07)	(0.07)	(0.06)	(0.08)	(0.07)	(0.07)	(0.06)
38	**0.37**	**0.41**	**0.41**	**0.40**	**0.36**	**0.39**	**0.40**	**0.39**
	(0.06)	(0.07)	(0.07)	(0.06)	(0.06)	(0.07)	(0.07)	(0.06)
39	**0.44**	**0.42**	**0.42**	**0.41**	**0.42**	**0.41**	**0.41**	**0.39**
	(0.07)	(0.07)	(0.06)	(0.06)	(0.07)	(0.06)	(0.06)	(0.06)
40	**0.45**	**0.44**	**0.41**	**0.41**	**0.44**	**0.42**	**0.40**	**0.39**
	(0.07)	(0.06)	(0.06)	(0.06)	(0.07)	(0.06)	(0.06)	(0.06)
41	**0.42**	**0.41**	**0.41**	**0.40**	**0.42**	**0.40**	**0.40**	**0.39**
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
42	**0.37**	**0.39**	**0.40**	**0.41**	**0.36**	**0.38**	**0.39**	**0.39**
	(0.07)	(0.07)	(0.06)	(0.06)	(0.07)	(0.07)	(0.07)	(0.06)
43	**0.39**	**0.38**	**0.39**	**0.40**	**0.36**	**0.36**	**0.38**	**0.39**
	(0.08)	(0.07)	(0.07)	(0.06)	(0.08)	(0.07)	(0.07)	(0.06)
44	**0.38**	**0.39**	**0.39**	**0.40**	**0.36**	**0.37**	**0.38**	**0.38**
	(0.07)	(0.07)	(0.07)	(0.06)	(0.08)	(0.07)	(0.07)	(0.06)
45	**0.4**	**0.40**	**0.40**	**0.40**	**0.39**	**0.38**	**0.38**	**0.39**
	(0.07)	(0.07)	(0.07)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)
46	**0.42**	**0.42**	**0.41**	**0.41**	**0.40**	**0.39**	**0.40**	**0.39**
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
47	**0.42**	**0.43**	**0.43**	**0.42**	**0.40**	**0.41**	**0.41**	**0.40**
	(0.07)	(0.7)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
48	**0.44**	**0.43**	**0.43**	**0.43**	**0.42**	**0.41**	**0.41**	**0.40**
	(0.08)	(0.07)	(0.07)	(0.07)	(0.08)	(0.07)	(0.07)	(0.07)
49	**0.44**	**0.44**	**0.43**	**0.43**	**0.42**	**0.42**	**0.41**	**0.41**
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.7)	(0.07)
50	**0.43**	**0.43**	**0.43**	**0.43**	**0.40**	**0.41**	**0.41**	**0.41**
	(0.07)	(0.07)	(0.08)	(0.07)	(0.07)	(0.07)	(0.7)	(0.07)
51	**0.42**	**0.42**	**0.43**	**0.43**	**0.40**	**0.40**	**0.41**	**0.41**
	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.8)	(0.07)
52	**0.43**	**0.43**	**0.43**	**0.43**	**0.41**	**0.41**	**0.41**	**0.41**
	(0.09)	(0.08)	(0.08)	(0.08)	(0.09)	(0.08)	(0.8)	(0.08)
53	**0.44**	**0.44**	**0.43**	-	**0.42**	**0.42**	**0.41**	-
	(0.08)	(0.08)	(0.08)		(0.08)	(0.08)	(0.8)	
54	**0.44**	**0.44**	-	-	**0.42**	**0.42**	-	-
	(0.08)	(0.08)			(0.08)	(0.08)		
55	**0.44**	-	-	-	**0.42**	-	-	-
	(0.07)				(0.07)			

The table presents age-specific estimates of the IGE of children's expected family income when they were between 26 and 55 years old. Point estimates are in bold, standard errors are in parentheses. Parental income is parents' average income when their children were 15-17 years old. The four columns in each panel differ on whether exact ages or three different age intervals are used to select the observations in each estimation sample (the values in the first column are the exact ages or the mid-points of the age intervals). A quadratic polynomial on the average age of parents when their children were 15-17 years old is used to control for parental age in models that do so.

Table E2: IGE of sons' expected earnings, by sons' age

Sons' age	Without controlling for parental age: Exact age	Three-year interval	Five-year interval	Seven-year interval	Controlling for parental age: Exact age	Three-year interval	Five-year interval	Seven-year interval
26	**0.16**	-	-	-	**0.15**	-	-	-
	(0.05)				(0.06)			
27	**0.15**	**0.16**	-	-	**0.16**	**0.16**	-	-
	(0.06)	(0.05)			(0.06)	(0.05)		
28	**0.16**	**0.16**	**0.16**	-	**0.17**	**0.16**	**0.16**	-
	(0.05)	(0.05)	(0.05)		(0.06)	(0.06)	(0.06)	
29	**0.16**	**0.17**	**0.19**	**0.20**	**0.15**	**0.17**	**0.19**	**0.20**
	(0.06)	(0.06)	(0.06)	(0.05)	(0.07)	(0.06)	(0.06)	(0.06)
30	**0.19**	**0.21**	**0.22**	**0.23**	**0.18**	**0.20**	**0.22**	**0.22**
	(0.07)	(0.07)	(0.06)	(0.06)	(0.08)	(0.07)	(0.07)	(0.06)
31	**0.27**	**0.25**	**0.25**	**0.24**	**0.27**	**0.25**	**0.24**	**0.24**
	(0.08)	(0.07)	(0.07)	(0.06)	(0.09)	(0.08)	(0.07)	(0.07)
32	**0.30**	**0.29**	**0.27**	**0.26**	**0.30**	**0.29**	**0.27**	**0.26**
	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.07)
33	**0.31**	**0.29**	**0.29**	**0.29**	**0.30**	**0.29**	**0.30**	**0.30**
	(0.09)	(0.08)	(0.07)	(0.07)	(0.09)	(0.08)	(0.08)	(0.07)
34	**0.25**	**0.30**	**0.31**	**0.32**	**0.27**	**0.31**	**0.32**	**0.33**
	(0.08)	(0.08)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)
35	**0.34**	**0.32**	**0.33**	**0.33**	**0.35**	**0.33**	**0.34**	**0.34**
	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
36	**0.36**	**0.36**	**0.34**	**0.35**	**0.37**	**0.37**	**0.36**	**0.36**
	(0.08)	(0.07)	(0.08)	(0.08)	(0.09)	(0.08)	(0.08)	(0.09)
37	**0.39**	**0.37**	**0.38**	**0.38**	**0.39**	**0.38**	**0.39**	**0.39**
	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)
38	**0.37**	**0.40**	**0.40**	**0.40**	**0.38**	**0.41**	**0.41**	**0.41**
	(0.11)	(0.10)	(0.10)	(0.09)	(0.11)	(0.10)	(0.10)	(0.09)
39	**0.44**	**0.42**	**0.42**	**0.41**	**0.44**	**0.43**	**0.43**	**0.42**
	(0.12)	(0.11)	(0.10)	(0.10)	(0.12)	(0.11)	(0.10)	(0.10)
40	**0.45**	**0.45**	**0.42**	**0.41**	**0.46**	**0.46**	**0.43**	**0.42**
	(0.11)	(0.11)	(0.11)	(0.10)	(0.11)	(0.11)	(0.11)	(0.10)
41	**0.45**	**0.43**	**0.43**	**0.41**	**0.46**	**0.44**	**0.43**	**0.42**
	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
42	**0.39**	**0.41**	**0.41**	**0.42**	**0.39**	**0.42**	**0.42**	**0.42**
	(0.11)	(0.10)	(0.10)	(0.10)	(0.11)	(0.10)	(0.10)	(0.10)
43	**0.40**	**0.39**	**0.41**	**0.43**	**0.41**	**0.40**	**0.41**	**0.43**
	(0.09)	(0.10)	(0.10)	(0.09)	(0.10)	(0.10)	(0.10)	(0.09)
44	**0.39**	**0.40**	**0.42**	**0.43**	**0.39**	**0.41**	**0.42**	**0.43**
	(0.10)	(0.09)	(0.09)	(0.09)	(0.10)	(0.10)	(0.09)	(0.09)
45	**0.43**	**0.43**	**0.43**	**0.43**	**0.42**	**0.43**	**0.43**	**0.43**
	(0.09)	(0.09)	(0.09)	(0.09)	(0.10)	(0.09)	(0.09)	(0.09)
46	**0.47**	**0.45**	**0.44**	**0.43**	**0.46**	**0.45**	**0.44**	**0.43**
	(0.09)	(0.09)	(0.09)	(0.08)	(0.09)	(0.09)	(0.09)	(0.08)
47	**0.46**	**0.46**	**0.45**	**0.43**	**0.45**	**0.46**	**0.44**	**0.43**
	(0.09)	(0.09)	(0.08)	(0.08)	(0.09)	(0.09)	(0.08)	(0.08)
48	**0.46**	**0.45**	**0.44**	**0.42**	**0.45**	**0.44**	**0.43**	**0.42**
	(0.10)	(0.09)	(0.08)	(0.08)	(0.10)	(0.09)	(0.08)	(0.08)
49	**0.43**	**0.42**	**0.41**	**0.41**	**0.43**	**0.42**	**0.41**	**0.41**
	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
50	**0.37**	**0.38**	**0.39**	**0.40**	**0.37**	**0.38**	**0.39**	**0.40**
	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
51	**0.30**	**0.36**	**0.38**	**0.39**	**0.34**	**0.35**	**0.37**	**0.39**
	(0.09)	(0.09)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)	(0.08)
52	**0.36**	**0.36**	**0.37**	**0.39**	**0.35**	**0.35**	**0.37**	**0.38**
	(0.11)	(0.10)	(0.09)	(0.08)	(0.11)	(0.10)	(0.09)	(0.08)
53	**0.37**	**0.38**	**0.38**	-	**0.37**	**0.38**	**0.37**	-
	(0.10)	(0.09)	(0.09)		(0.10)	(0.10)	(0.09)	
54	**0.40**	**0.40**	-	-	**0.41**	**0.40**	-	-
	(0.08)	(0.09)			(0.09)	(0.09)		
55	**0.43**	-	-	-	**0.42**	-	-	-
	(0.09)				(0.09)			

The table presents age-specific estimates of the IGE of sons' expected earnings when they were between 26 and 55 years old. Point estimates are in bold, standard errors are in parentheses. Parental income is parents' average income when their children were 15-17 years old. The four columns in each panel differ on wether exact ages or three different age intervals are used to select the observations in each estimation sample (the values in the first column are either the exact ages or the mid-points of the age intervals). A quadratic polynomial on the average age of parents when their children were 15-17 years old is used to control for parental age in models that do so.

Figure E3: IGE of children's expected family income, by number of years of parental information and parental age at time of measurement

Children's age around which parental measure is centered	Parents' average age	Number of years of parental information												
		1	3	5	7	9	11	13	15	17	19	21	23	25
1	28.7	**0.25**	-	-	-	-	-	-	-	-	-	-	-	-
		(0.10)												
2	29.7	**0.43**	**0.48**	-	-	-	-	-	-	-	-	-	-	-
		(0.09)	(0.10)											
3	30.6	**0.34**	**0.52**	**0.55**	-	-	-	-	-	-	-	-	-	-
		(0.10)	(0.09)	(0.10)										
4	31.6	**0.37**	**0.54**	**0.58**	**0.60**	-	-	-	-	-	-	-	-	-
		(0.12)	(0.09)	(0.10)	(0.10)									
5	32.5	**0.46**	**0.57**	**0.59**	**0.61**	**0.62**	-	-	-	-	-	-	-	-
		(0.10)	(0.09)	(0.10)	(0.10)	(0.10)								
6	33.5	**0.45**	**0.58**	**0.60**	**0.61**	**0.62**	**0.63**	-	-	-	-	-	-	-
		(0.11)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)							
7	34.5	**0.41**	**0.57**	**0.60**	**0.61**	**0.61**	**0.63**	**0.63**	-	-	-	-	-	-
		(0.11)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)						
8	35.5	**0.47**	**0.55**	**0.58**	**0.60**	**0.61**	**0.61**	**0.62**	**0.63**	-	-	-	-	-
		(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)					
9	36.5	**0.38**	**0.54**	**0.57**	**0.59**	**0.60**	**0.61**	**0.62**	**0.62**	**0.62**	-	-	-	-
		(0.10)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)				
10	37.5	**0.40**	**0.50**	**0.56**	**0.57**	**0.59**	**0.60**	**0.60**	**0.61**	**0.62**	**0.62**	-	-	-
		(0.08)	(0.09)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)			
11	38.5	**0.33**	**0.48**	**0.54**	**0.56**	**0.58**	**0.58**	**0.60**	**0.60**	**0.61**	**0.61**	**0.61**	-	-
		(0.10)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)		
12	39.4	**0.44**	**0.50**	**0.52**	**0.55**	**0.56**	**0.58**	**0.59**	**0.60**	**0.60**	**0.60**	**0.61**	**0.61**	-
		(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	
13	40.4	**0.31**	**0.48**	**0.52**	**0.53**	**0.55**	**0.57**	**0.58**	**0.58**	**0.59**	**0.59**	**0.60**	**0.60**	**0.61**
		(0.08)	(0.06)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
14	41.4	**0.38**	**0.45**	**0.50**	**0.53**	**0.55**	**0.56**	**0.57**	**0.57**	**0.58**	**0.59**	**0.59**	**0.60**	-
		(0.07)	(0.08)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	
15	42.4	**0.31**	**0.44**	**0.51**	**0.53**	**0.54**	**0.54**	**0.55**	**0.56**	**0.57**	**0.58**	**0.58**	-	-
		(0.11)	(0.08)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)		
16	43.4	**0.31**	**0.49**	**0.51**	**0.52**	**0.53**	**0.53**	**0.54**	**0.55**	**0.56**	**0.57**	-	-	-
		(0.08)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)			
17	44.3	**0.37**	**0.44**	**0.51**	**0.51**	**0.51**	**0.53**	**0.53**	**0.54**	**0.55**	-	-	-	-
		(0.09)	(0.08)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)				
18	45.3	**0.28**	**0.48**	**0.48**	**0.50**	**0.51**	**0.52**	**0.52**	**0.53**	-	-	-	-	-
		(0.10)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)					
19	46.3	**0.36**	**0.45**	**0.48**	**0.48**	**0.50**	**0.51**	**0.52**	-	-	-	-	-	-
		(0.08)	(0.06)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)						
20	47.3	**0.20**	**0.40**	**0.46**	**0.48**	**0.49**	**0.50**	-	-	-	-	-	-	-
		(0.09)	(0.07)	(0.06)	(0.06)	(0.06)	(0.07)							
21	48.3	**0.34**	**0.41**	**0.45**	**0.46**	**0.49**	-	-	-	-	-	-	-	-
		(0.07)	(0.06)	(0.06)	(0.06)	(0.06)								
22	49.3	**0.27**	**0.43**	**0.43**	**0.46**	-	-	-	-	-	-	-	-	-
		(0.09)	(0.06)	(0.06)	(0.06)									
23	50.3	**0.09**	**0.41**	**0.45**	-	-	-	-	-	-	-	-	-	-
		(0.12)	(0.06)	(0.06)										
24	51.3	**0.36**	**0.40**	-	-	-	-	-	-	-	-	-	-	-
		(0.05)	(0.06)											
25	52.3	**0.27**	-	-	-	-	-	-	-	-	-	-	-	-
		(0.09)												

The table presents estimates of the IGE of children's expected family income based on parental income measures that differ on the number of years of parental information used to compute them, and on the parents' average age in those years. Point estimates are in bold, standard errors are in parentheses. Parental income is measured around the children's age indicated in the first column, when the parents' average age is as indicated in the second column. The other columns show IGE estimates based on 1, 3, …, 23, 25 years of parental information. Children's family income is measured at ages 36-38.

Figure E4: IGE of sons' expected earnings, by number of years of parental information and parental age at time of measurement

Sons' age around which parental measure is centered	Parents' average age	Number of years of parental informatino												
		1	3	5	7	9	11	13	15	17	19	21	23	25
1	28.6	**0.23**	-	-	-	-	-	-	-	-	-	-	-	-
		(0.11)												
2	29.5	**0.45**	**0.54**	-	-	-	-	-	-	-	-	-	-	-
		(0.10)	(0.10)											
3	30.5	**0.29**	**0.59**	**0.63**	-	-	-	-	-	-	-	-	-	-
		(0.15)	(0.09)	(0.09)										
4	31.4	**0.35**	**0.63**	**0.64**	**0.65**	-	-	-	-	-	-	-	-	-
		(0.15)	(0.08)	(0.09)	(0.09)									
5	32.4	**0.49**	**0.61**	**0.64**	**0.67**	**0.67**	-	-	-	-	-	-	-	-
		(0.14)	(0.08)	(0.09)	(0.09)	(0.09)								
6	33.4	**0.47**	**0.61**	**0.64**	**0.65**	**0.66**	**0.68**	-	-	-	-	-	-	-
		(0.11)	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)							
7	34.4	**0.45**	**0.58**	**0.62**	**0.63**	**0.66**	**0.68**	**0.70**	-	-	-	-	-	-
		(0.10)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)						
8	35.4	**0.45**	**0.55**	**0.59**	**0.64**	**0.65**	**0.68**	**0.70**	**0.71**	-	-	-	-	-
		(0.10)	(0.08)	(0.09)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)					
9	36.4	**0.40**	**0.55**	**0.59**	**0.62**	**0.66**	**0.67**	**0.69**	**0.71**	**0.73**	-	-	-	-
		(0.09)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)	(0.08)				
10	37.4	**0.39**	**0.53**	**0.59**	**0.62**	**0.64**	**0.67**	**0.69**	**0.72**	**0.74**	**0.75**	-	-	-
		(0.09)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)			
11	38.4	**0.25**	**0.53**	**0.59**	**0.61**	**0.64**	**0.66**	**0.70**	**0.72**	**0.74**	**0.73**	**0.72**	-	-
		(0.15)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)		
12	39.4	**0.52**	**0.58**	**0.59**	**0.61**	**0.64**	**0.68**	**0.70**	**0.72**	**0.71**	**0.70**	**0.70**	**0.71**	-
		(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)	
13	40.3	**0.30**	**0.58**	**0.60**	**0.62**	**0.66**	**0.68**	**0.70**	**0.69**	**0.69**	**0.69**	**0.70**	**0.71**	**0.72**
		(0.12)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)
14	41.3	**0.38**	**0.47**	**0.61**	**0.66**	**0.67**	**0.69**	**0.68**	**0.67**	**0.67**	**0.68**	**0.69**	**0.71**	-
		(0.12)	(0.13)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	
15	42.3	**0.29**	**0.49**	**0.64**	**0.67**	**0.68**	**0.66**	**0.65**	**0.65**	**0.67**	**0.68**	**0.69**	-	-
		(0.14)	(0.13)	(0.07)	(0.07)	(0.07)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)		
16	43.3	**0.26**	**0.61**	**0.65**	**0.67**	**0.65**	**0.64**	**0.64**	**0.65**	**0.66**	**0.67**	-	-	-
		(0.12)	(0.06)	(0.07)	(0.07)	(0.08)	(0.08)	(0.09)	(0.08)	(0.08)	(0.08)			
17	44.2	**0.34**	**0.47**	**0.65**	**0.63**	**0.62**	**0.63**	**0.64**	**0.65**	**0.66**	-	-	-	-
		(0.12)	(0.14)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)				
18	45.2	**0.40**	**0.61**	**0.59**	**0.59**	**0.60**	**0.62**	**0.63**	**0.64**	-	-	-	-	-
		(0.13)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)					
19	46.2	**0.43**	**0.54**	**0.55**	**0.57**	**0.60**	**0.61**	**0.63**	-	-	-	-	-	-
		(0.12)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)						
20	47.2	**0.26**	**0.45**	**0.53**	**0.56**	**0.58**	**0.60**	-	-	-	-	-	-	-
		(0.11)	(0.08)	(0.08)	(0.08)	(0.08)	(0.07)							
21	48.2	**0.37**	**0.43**	**0.51**	**0.55**	**0.57**	-	-	-	-	-	-	-	-
		(0.06)	(0.07)	(0.07)	(0.07)	(0.07)								
22	49.2	**0.29**	**0.47**	**0.48**	**0.52**	-	-	-	-	-	-	-	-	-
		(0.09)	(0.06)	(0.07)	(0.07)									
23	50.2	**0.43**	**0.47**	**0.49**	-	-	-	-	-	-	-	-	-	-
		(0.06)	(0.06)	(0.06)										
24	51.2	**0.43**	**0.48**	-	-	-	-	-	-	-	-	-	-	-
		(0.07)	(0.06)											
25	52.2	**0.39**	-	-	-	-	-	-	-	-	-	-	-	-
		(0.06)												

The table presents estimates of the IGE of sons' expected earnings based on parental income measures that differ on the number of years of parental information used to compute them, and on the parents' average age in those years. Point estimates are in bold, standard errors are in parentheses. Parental income is measured around the children's age indicated in the first column, when the parents' average age is as indicated in the second column. The other columns show IGE estimates based on 1, 3, …, 23, 25 years of parental information. Children's earnings is measured at ages 36-38.

Table E5: IGE of children's expected family income, by children's age and number of years of parental information

Number of years of parental information	Without controlling for parental age					Controlling for parental age				
	Children's age					Children's age				
	24-26	27-29	30-32	33-35	36-38	24-26	27-29	30-32	33-35	36-38
1	**0.18**	**0.22**	**0.27**	**0.29**	**0.31**	**0.17**	**0.20**	**0.24**	**0.27**	**0.29**
	(0.04)	(0.05)	(0.05)	(0.07)	(0.08)	(0.04)	(0.05)	(0.05)	(0.07)	(0.08)
3	**0.29**	**0.35**	**0.41**	**0.45**	**0.48**	**0.29**	**0.33**	**0.40**	**0.43**	**0.48**
	(0.03)	(0.04)	(0.04)	(0.05)	(0.06)	(0.03)	(0.05)	(0.04)	(0.05)	(0.07)
5	**0.30**	**0.36**	**0.44**	**0.47**	**0.52**	**0.29**	**0.35**	**0.42**	**0.46**	**0.52**
	(0.03)	(0.04)	(0.04)	(0.05)	(0.07)	(0.03)	(0.05)	(0.04)	(0.06)	(0.07)
7	**0.30**	**0.37**	**0.44**	**0.48**	**0.53**	**0.30**	**0.35**	**0.43**	**0.46**	**0.53**
	(0.03)	(0.04)	(0.04)	(0.06)	(0.07)	(0.03)	(0.05)	(0.05)	(0.06)	(0.07)
9	**0.32**	**0.39**	**0.46**	**0.50**	**0.55**	**0.31**	**0.37**	**0.45**	**0.49**	**0.56**
	(0.03)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)
11	**0.33**	**0.40**	**0.47**	**0.52**	**0.57**	**0.32**	**0.39**	**0.46**	**0.51**	**0.57**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)
13	**0.34**	**0.41**	**0.48**	**0.53**	**0.58**	**0.33**	**0.40**	**0.47**	**0.52**	**0.59**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)
15	**0.34**	**0.42**	**0.49**	**0.54**	**0.58**	**0.34**	**0.41**	**0.48**	**0.53**	**0.59**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)
17	**0.35**	**0.43**	**0.49**	**0.55**	**0.59**	**0.34**	**0.41**	**0.48**	**0.54**	**0.60**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)
19	**0.35**	**0.43**	**0.50**	**0.56**	**0.59**	**0.35**	**0.42**	**0.49**	**0.55**	**0.60**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)
21	**0.36**	**0.44**	**0.51**	**0.56**	**0.60**	**0.35**	**0.43**	**0.50**	**0.56**	**0.61**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)
23	**0.36**	**0.45**	**0.51**	**0.57**	**0.60**	**0.36**	**0.43**	**0.50**	**0.56**	**0.61**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)
25	**0.37**	**0.45**	**0.52**	**0.58**	**0.61**	**0.36**	**0.44**	**0.51**	**0.57**	**0.62**
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)

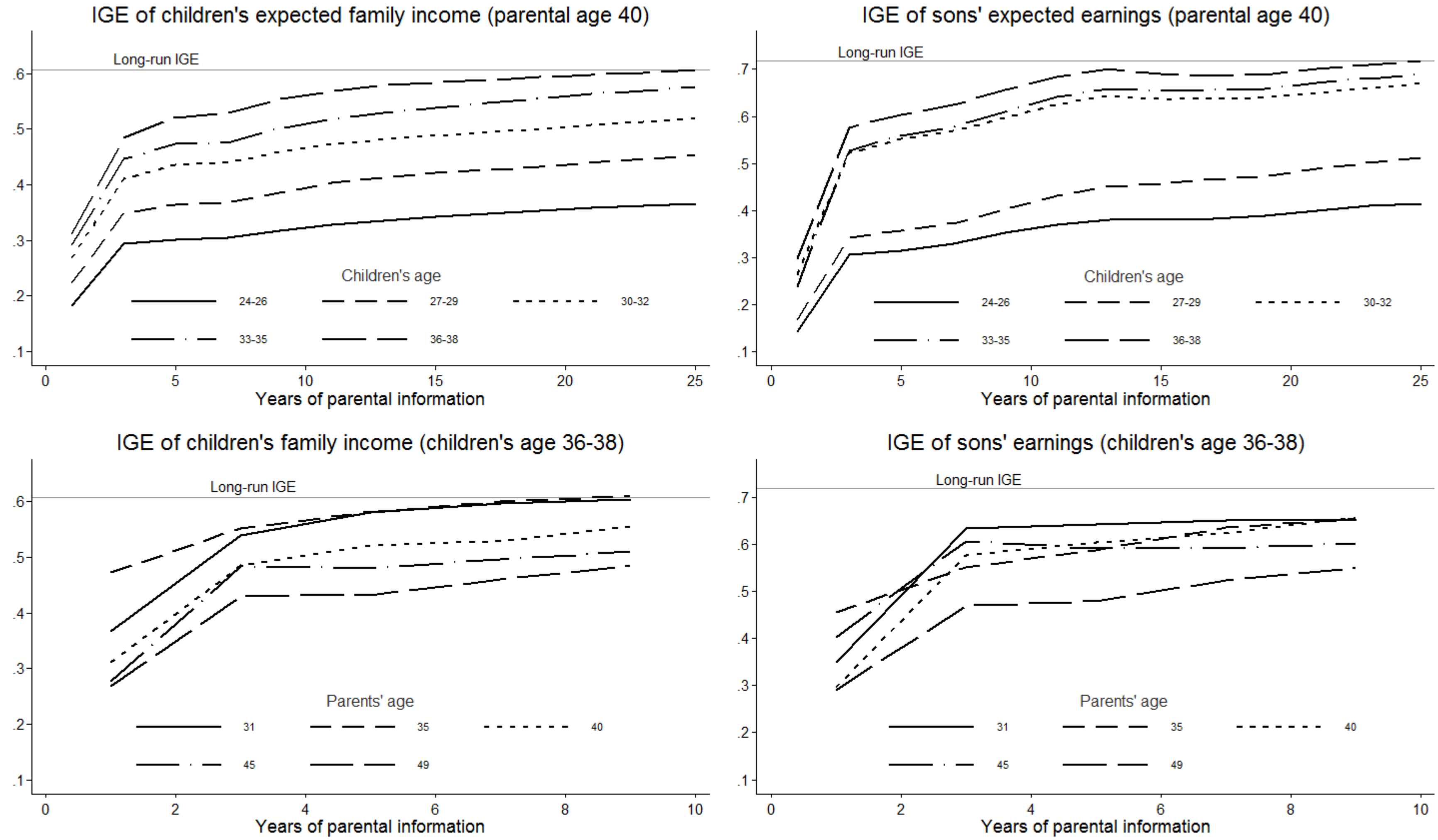
The table presents estimates of the IGE of children's expected family income by children's age and number of years of parental information. Point estimates are in bold, standard errors are in parentheses. Parental income is parents' average income over the number of years indicated in the first column, with the period centered at the year the children were 13 years old (when parents' average age is about 40). Children's income is their family income at the ages indicated at the top of each column. A quadratic polynomial on the average age of parents when their income is measured is used to control for parental age in models that do so.

Table E6: IGE of sons' expected earnings, by sons' age and number of years of parental information

Number of years of parental information	Without controlling for parental age					Controlling for parental age				
	Sons' age					Sons' age				
	24-26	27-29	30-32	33-35	36-38	24-26	27-29	30-32	33-35	36-38
1	**0.14**	**0.17**	**0.26**	**0.24**	**0.30**	**0.12**	**0.12**	**0.22**	**0.20**	**0.26**
	(0.05)	(0.07)	(0.10)	(0.11)	(0.12)	(0.04)	(0.06)	(0.09)	(0.09)	(0.11)
3	**0.31**	**0.34**	**0.52**	**0.53**	**0.58**	**0.28**	**0.27**	**0.48**	**0.50**	**0.55**
	(0.05)	(0.11)	(0.07)	(0.07)	(0.07)	(0.05)	(0.12)	(0.07)	(0.07)	(0.07)
5	**0.31**	**0.36**	**0.55**	**0.56**	**0.60**	**0.29**	**0.29**	**0.52**	**0.53**	**0.58**
	(0.05)	(0.11)	(0.07)	(0.07)	(0.07)	(0.05)	(0.13)	(0.07)	(0.08)	(0.07)
7	**0.33**	**0.37**	**0.57**	**0.58**	**0.62**	**0.31**	**0.31**	**0.53**	**0.55**	**0.60**
	(0.05)	(0.11)	(0.07)	(0.08)	(0.07)	(0.05)	(0.13)	(0.07)	(0.08)	(0.08)
9	**0.35**	**0.40**	**0.60**	**0.61**	**0.66**	**0.33**	**0.34**	**0.56**	**0.58**	**0.63**
	(0.05)	(0.10)	(0.07)	(0.08)	(0.07)	(0.05)	(0.12)	(0.08)	(0.08)	(0.08)
11	**0.37**	**0.43**	**0.63**	**0.64**	**0.68**	**0.35**	**0.37**	**0.59**	**0.62**	**0.66**
	(0.05)	(0.10)	(0.08)	(0.08)	(0.08)	(0.05)	(0.11)	(0.08)	(0.08)	(0.08)
13	**0.38**	**0.45**	**0.64**	**0.66**	**0.70**	**0.36**	**0.39**	**0.61**	**0.64**	**0.68**
	(0.06)	(0.10)	(0.08)	(0.08)	(0.08)	(0.05)	(0.11)	(0.08)	(0.08)	(0.08)
15	**0.38**	**0.46**	**0.64**	**0.65**	**0.69**	**0.36**	**0.39**	**0.60**	**0.63**	**0.67**
	(0.06)	(0.09)	(0.09)	(0.09)	(0.08)	(0.06)	(0.10)	(0.09)	(0.09)	(0.09)
17	**0.38**	**0.47**	**0.64**	**0.66**	**0.69**	**0.37**	**0.40**	**0.61**	**0.64**	**0.67**
	(0.06)	(0.09)	(0.09)	(0.09)	(0.09)	(0.06)	(0.10)	(0.09)	(0.09)	(0.09)
19	**0.39**	**0.47**	**0.64**	**0.66**	**0.69**	**0.37**	**0.41**	**0.61**	**0.64**	**0.67**
	(0.06)	(0.09)	(0.10)	(0.10)	(0.09)	(0.06)	(0.10)	(0.10)	(0.10)	(0.09)
21	**0.40**	**0.49**	**0.65**	**0.67**	**0.70**	**0.39**	**0.43**	**0.62**	**0.65**	**0.69**
	(0.07)	(0.09)	(0.09)	(0.09)	(0.09)	(0.06)	(0.10)	(0.10)	(0.10)	(0.09)
23	**0.41**	**0.50**	**0.66**	**0.68**	**0.71**	**0.39**	**0.44**	**0.63**	**0.66**	**0.70**
	(0.07)	(0.08)	(0.09)	(0.09)	(0.09)	(0.07)	(0.09)	(0.10)	(0.10)	(0.09)
25	**0.42**	**0.51**	**0.67**	**0.69**	**0.72**	**0.40**	**0.45**	**0.64**	**0.67**	**0.71**
	(0.06)	(0.08)	(0.09)	(0.09)	(0.09)	(0.07)	(0.09)	(0.09)	(0.10)	(0.09)

The table presents estimates of the IGE of sons' expected earnings by sons' age and number of years of parental information. Point estimates are in bold, standard errors are in parentheses. Parental income is parents' average income over the number of years indicated in the first column, with the period centered at the year the sons were 13 years old (when parents' average age is about 40). Sons' earnings is their earnings at the ages indicated at the top of each column. A quadratic polynomial on the average age of parents when their income is measured is used to control for parental age in models that do so.

Figure E1: IGE estimates by parental information and children's and parents' ages



The figure shows how the relationship between IGE estimates and the number of years of information used to compute parental income changes across children's and parents' ages. The curves display estimates of the IGE of expected children's income and sons' earnings at different children's ages (top panels) and parental ages (bottom panels). The measures of children's family income and sons' earnings pertain to three-year age intervals. The short-run measures of parental income are based on 1, 3, 5, 7 and 9 years of information. In the top panels these measures are centered around the year the children were 13 years old (when mean parental age is about 40). In the bottom panels children's family income and sons' earnings refer to when they were 36-38 years old, while parents' age is the average age of the parents in the sample. The (approximated) long-run IGEs are computed with parents' information pertaining to when their children were 1-25 years old, and with children's information pertaining to when they were 36-38 years old. Two points in each of the bottom panels were computed by extrapolation.